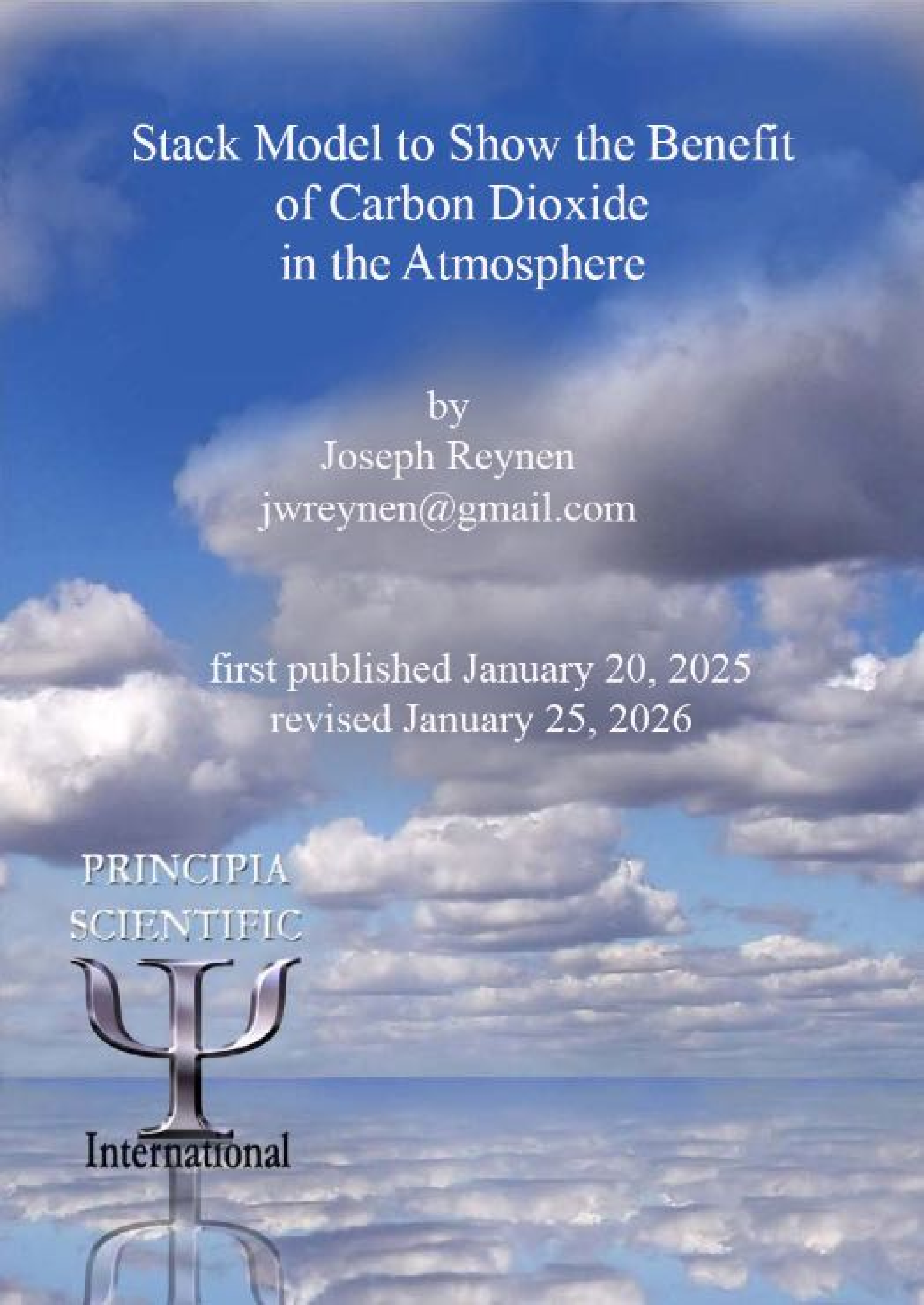

Stack Model to Show the Benefit of Carbon Dioxide in the Atmosphere

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Summary

Today in the United States of America the 47th President has been installed. One of the promises of the new President is to withdraw the US from the 2015 Paris agreement on Climate Change, for the second time.
It is to be expected that this time many other counties will follow the US.
The author has been skeptic on Climate Change for more than a decade, in particular concerning the claim of the IPCC (International Panel on Climate Change) that CO_2 gas causes the planet Earth to heat up.
There is not such a climate crises, it is a hoax. CO_2 is plant food.
As a retired scientist, in the field of safety of nuclear reactors, the author has used the finite element method (FEM) to analyze the LW (long wave) radiation from the surface of the planet to send back to outer space the heat which the sun is sending by SW (short wave) radiation to planet Earth.
The infra-red-active trace gases are simulated by a stack of fine gauze.
This paper with the **Appendices** repeats the basis of the stack model:

- phenomena of thermalization and saturation of carbon dioxide gas CO_2
- the one-stream formulation of Stefan-Boltzmann-Christiansen of 1883 for the LW radiation through water vapor and carbon dioxide gas, which are the infra-red-active gases considered.
- using the ideal gas law (pv = RT) to derive the natural adiabatic atmospheric temperature distribution with lapse rates of the so-called Standard Atmosphere, and the distribution of CO_2 gas in such atmosphere up to a height of 30 km, on the basis of Newton gravitation law.
- listing of the (.m) files of the stack model which can be used on any PC with an installed MATLAB program.

This material can be used by any person with a science middle school education, or better with the mathematics of first year science faculties.

History of the stack model to analyze the evacuation of heat from the planet.

Infra-red-active gases in the atmosphere are mostly gases with 3 or more atoms per molecule: water-vapor (H_2O), carbon dioxide (CO_2), methane (CH_4), ozone (O_3), laughing gas (N_2O), ...
They are also known as greenhouse gases, a misnomer!
Infra-red-active gases hinder long-wave (LW) radiation to outer space from the surface of the planet, to evacuate the heat which the sun is sending to the planet as short-wave (SW) radiation with smaller absorption losses in aerosols.
The most important infra-red-active gases are H_2O vapor and CO_2 gas: they are the subject of this paper.
In earlier papers **[1]** the author has studied the hindering of the LW evacuation of heat from the terrestrial surface to outer-space by a mechanism of a stack of fine gauze, simulating the infra-red-active gases.
A simple model but extremely transparent and efficient.

The stack model analyzes the long-wave (LW) heat transport through a stack of absorbing grids in a column of 30 km high.
A grid(i) is characterized by the height z(i) and an absorption coefficient f(i), much smaller than 1.
The coefficient f(i) represents the ratio of the cross-section of the wires of the grid(i) with respect to the total surface area of the grid(i).
The layers have given temperatures in degrees Kelvin, T(i) [ºK].
We define ftot = Σf(i), it represents the total absorption of the gids in the stack, ftotH2O in the water model and ftotCO2 in the carbon dioxide model. The heat flux φ between two grids in W/m² becomes:

$$\varphi(i \rightarrow j) = fij*\sigma*(T(i)^4 - T(j)^4) \quad \text{for } T(i) > T(j) \quad \text{and} \quad \varphi(j \rightarrow i) = 0 \quad \textbf{(1)}$$

$$\text{With } \sigma = 5.67e\text{-}8 \ W/m^2/^\circ K^4 \quad \text{and} \quad 1/fij = 1/f(i) + 1/f(j) - 1$$

Relation **(1)** is the classic Stefan-Boltzmann-Christiansen relation for the one-stream formulation of the energy transport by radiation between the wire of grid(i) with cross-section f(i) and temperature T(i), and the wire of grid(j) with cross-section f(j) and temperature T(j). See **Appendix 1**

Finite Element Method (FEM) techniques have been applied to generate a great number of simultaneous algebraic equations describing the heat transport in the atmosphere for both the H_2O model and the CO_2 model.

It has turned out:

- when the f(i) coefficients are taken proportional to the variable density of H_2O vapor in the atmosphere and the measured values T(i) for the temperature, the LW heat transport in a stack of 50 grids over a height of 11.5 km corresponds to results by main-stream authors of an atmosphere with only traces of infra-red-active water vapor.
  Those comparisons have been made with so-called global K&T diagrams.

- when the variable density of CO_2 is used for f(i) and the measured values T(i) for the temperature in a stack of 100 grids over a height of 30 km, the LW heat transport corresponds to results of fundamental physics of an atmosphere with only traces of infra-red-active CO_2 gas.
  Comparisons are made with the Happer results **[3]**.

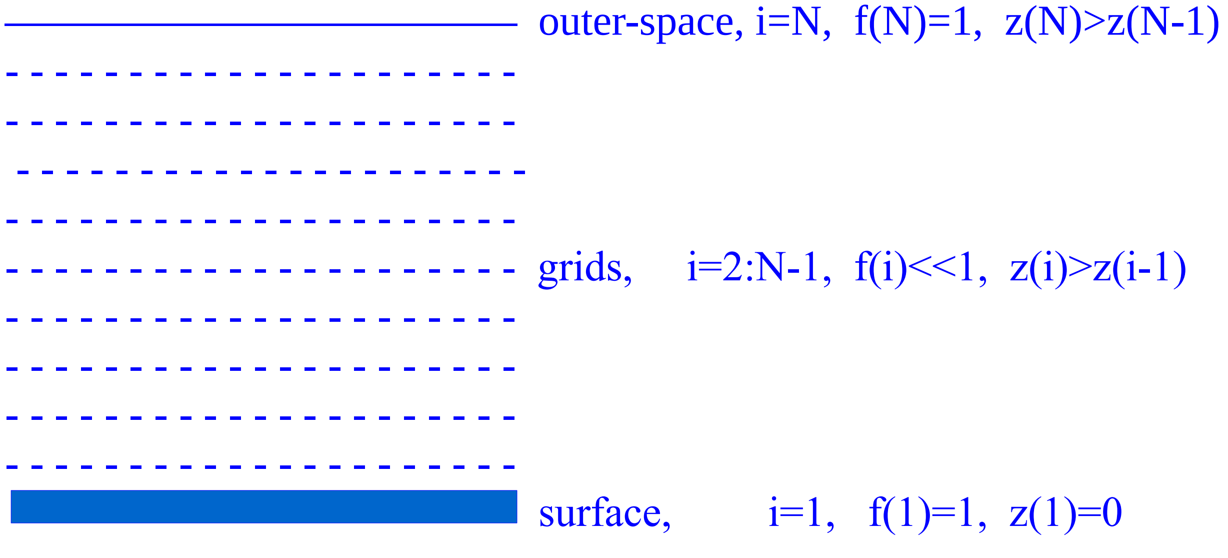
What is called *saturation* of CO_2 — for ftotCO2 > 1 (or > 400 ppm in a 30 km high stack) — has been demonstrated transparently with a to zero going "*window*“ for non-conventional *overlapping* finite elements.
The phenomenon *saturation* of CO_2 is often misinterpreted:
it is less correct to say “*CO_2 saturates at 400 ppm*”, one should rather say: *“LW radiation through CO_2 columns, with height of 30 km in the atmospheric gravitation field of the planet earth, becomes saturated at a concentration of 400 ppm CO_2”.*
The detailed atmospheric CO_2 density distribution follows from the *measured* atmospheric temperature distribution of the so-called Standard Atmosphere, taking into account the pressure decrease with height, defined by the Newton gravitation relation, discussed in **Appendix 2.**
The stack model of LW radiation can also include the phenomenon of *thermalization* of CO_2.
As argued by Pangburn **[2],** carbon dioxide molecules hit by LW radiation from the surface, transfer the absorbed energy before it has been re-emitted as LW radiation, by means of collision with molecules of the bulk of the atmosphere, nitrogen N_2, oxygen O_2 and the infra-red active H_2O.
The exchanged heat has lost its CO_2 identity and is re-radiated by the large concentration of H_2O molecules at H_2O frequencies.

The use of FEM in the stack model

The stack model is not using FEM in the classical way of solving differential equations, but rather using FEM strategies to model the phenomenon by generating a great number of simultaneous algebraic relations describing the LW heat transport, based on the Stefan-Boltzmann-Christiansen one-stream formulation of equation **(1).**

We give a short description of this non-conventional FEM approach.
We consider in **figure 1** a stack of N-2 grids, with dimensionless small absorption/emission coefficients f(i) << 1, being the ratio of the cross-section of the wires divided by the total surface.
The coefficients f(i) are assembled in a vector denoted by a bold character **f** of order N, including f(1) =1 for the surface and f(N) =1 for outer-space.
We define ftot =sum(**f**) - 2, being the sum for the atmospheric grids.
Consider in **figure 1** two layers i and j of black grids with coefficients f(i) and f(j) and absolute temperatures (Kelvin) T(i) and T(j), respectively.

Figure 1 Stack of fine gauze



The classical Stefan-Boltzmann-Christiansen relation **(1)** with

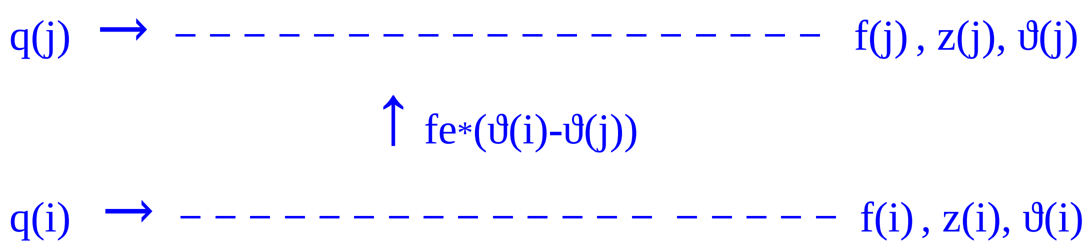
$$\vartheta = \sigma * T^4 \quad \text{and} \quad 1/fe = 1/fij = 1/f(i) + 1/f(j) - 1$$

can be written as:

$$\varphi(i \rightarrow j) = fe*(\vartheta(i)-\vartheta(j)) \qquad \textbf{(1a)}$$

Figure 2 shows the details of a simple radiation finite element.

Figure 2 Radiation finite element



Nodal parameters: f absorption coefficient

z	coordinate	[m]
ϑ	variable representing $\sigma^* T^4$	[W/m²]
q	external heat load into the grids	[W/m²]

Constitutive relation : fe element radiation coefficient

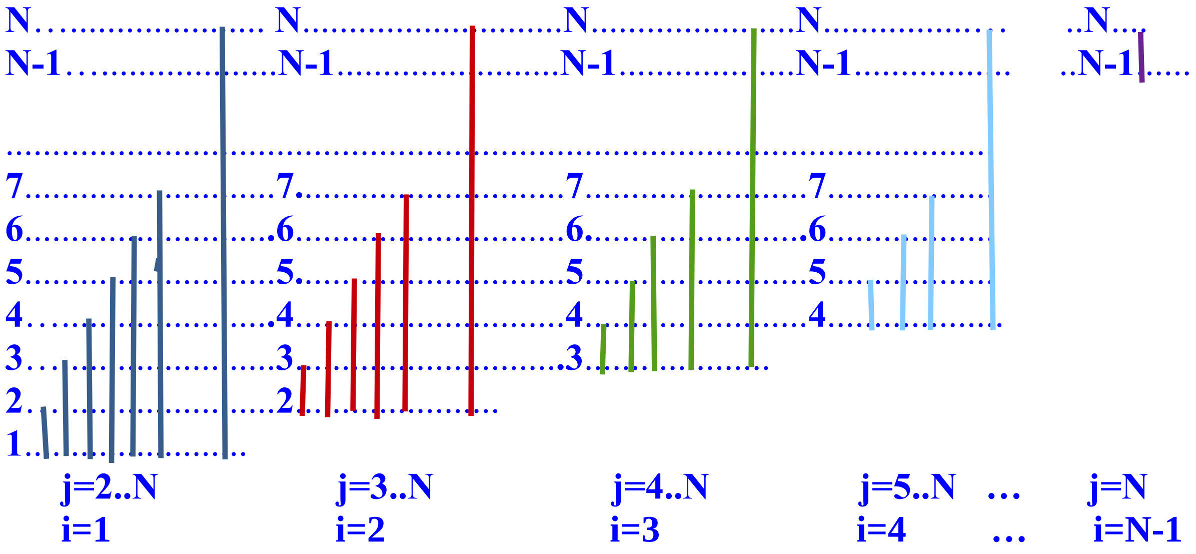
By means of a Galerkin-type of variation process, the heat balance of an element(i,j) can be written as:

$$\begin{vmatrix} q(i) \\ q(j) \end{vmatrix} = \begin{vmatrix} fe & -fe \\ -fe & fe \end{vmatrix} \begin{vmatrix} \vartheta(i) \\ \vartheta(j) \end{vmatrix} \qquad \textbf{(2)}$$

Equations **(2)** describe, for given ϑ(i) , ϑ(j) and fe, the flow of heat by LW radiation between grids i and j of an element(i , j) and the necessary external heat sources q(i) and q(j), for a balance.
Radiation elements can overlap with each other, as shown in **figure 3.**

Figure 3 Illustrative scheme of N(N-1)/2 overlapping finite elements (i , j)

All vertical lines represent an element to be squeezed into one column.



In a stack of nodes, with increasing height coordinates, any pair of two nodes i and j represents a finite element. For a stack of N nodes there are N(N-1)/2 finite elements with a characteristic heat balance like **(2).**

For an element(i ,j) with **adjacent** nodes ─ node i and node j = i+1 ─ the element transfer coefficient is defined by the Christiansen coefficient discussed in **Appendix1:**

$$1/fij = 1/f(i) + 1/f(j) - 1 \qquad \mathbf{(3)}$$

However, when between grid i and grid j of an element(i , j), other grids of other elements are present, the transfer of heat by LW radiation between grid i and grid j of element(i , j) will be hindered and fe of element (i,j) becomes:

$$fe = viewfactor(i\,,j) * fij \qquad \mathbf{(3a)}$$

In **(3a)** the viewfactor(i , j) takes into account the fact that other grids k are present between grid i and grid j of an element (i, j).
The viewfactor(i , j) between the nodes i and j of an element is defined by:

$$viewfactor(i\,,j) = 1 - \sum f(k) \qquad \text{for} \qquad z(i)<z(k)<z(j) \qquad \mathbf{(3b)}$$

The viewfactor(i , j) can be considered as a variable *open/closed window* between node i and node j. We have used the name *viewfactor,* since the name *window* is already used by mainstream authors on the subject as *viewfactor (1,N)*: between the surface of the planet and outer space.

The element matrices **(2)** for the N*(N-1)/2 pairs of nodes are assembled in a system matrix of order NxN, denominated by a bold character **K.**

The characteristic N equations of the atmospheric LW radiation become :

$$\mathbf{q} = \mathbf{K} * \boldsymbol{\vartheta} \qquad \mathbf{(4)}$$

The reader might appreciate the fact that each of the N nodes in the stack communicates with all the other N-1 nodes.

But the viewfactors limit the communication between nodes that are more than 1 grid apart. Indeed, the stack model with non-conventional overlapping elements represents reality.
For the two infra-red-active molecules of H_2O vapor and CO_2 gas, separate models are used, which in principle do not interfere which each other.
Each model defines an **O**utgoing **L**W **R**adiation flux, respectively OLR_{H20} and OLR_{CO2}, depending on the two concentrations of infra-red active gases, water vapor H_2O and carbon dioxide gas CO_2.
They share however common temperatures of atmosphere and surface.
The sum of the two OLR's is equal to the outgoing flux qtoa at the top of atmosphere of the planet equal to the global average of incoming SW radiation from the Sun: $OLR_{H20} + OLR_{CO2} = qtoa = 240\ W/m^2$
If for example OLR_{CO2} for some reason would decrease, OLR_{H2O} should increase in order to satisfy the energy balance of the planet.
OLR_{H2O} varies with the surface temperature TsK.
The stack H_2O model defines the derivative:

$$dOLR_{H2O}/dTsK = 3.2\ W/m^2/^oK. \qquad \textbf{(5)}$$

Relation **(5)** and the phenomenon thermalization connect the H_2O model with the CO_2 model.

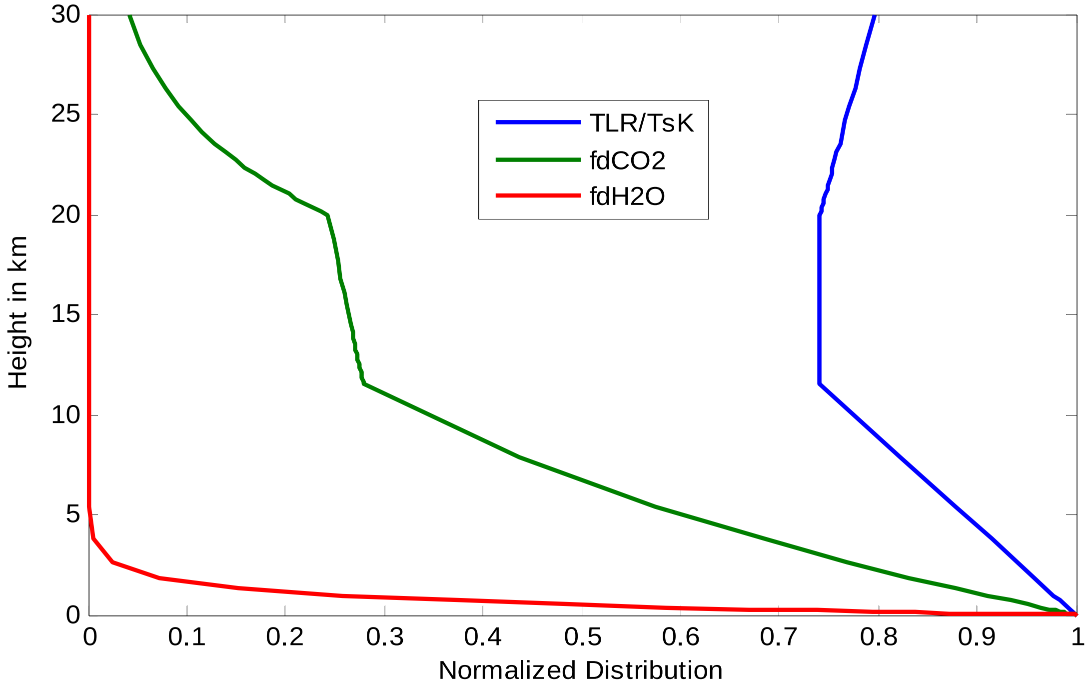
IPCC authors claim that for increasing CO_2 the planet temperature increases.
However the CO_2 stack model shows that for an increasing CO_2 concentration beyond 400 ppm the OLR_{CO2} does not decrease anymore: the 30 km high CO_2 column in the gravitation field of the planet earth is said to be saturated.

Data for the components of the vector ϑ and the matrix K

Data for the stack model are shown in **figure 4**:
measured temperature distribution and *normalized* concentration of water vapor H_2O and of carbon dioxide gas CO_2 over a height of 30 km.

Figure 4

fig 3.1 Standard normalized Temperature,TsK = 288.7209 and exponential decay of water vapor
CO2 distribution based on constant volumetric ppm,
ideal gas law p= rho*R*T, with standard atmosphere for temperature
and with gravitation law of Newton dp/dz = - rho*g



For a height up to 11.5 km the temperature is defined by the surface temperature and the environmental lapse rate, L = dT/dz = -6.5 K/km.

NB Main-stream papers define ***E****nvironmental* ***L****apse* ***R****ate ELR = - dT/dz.*
We prefer to work with a more logical definition L= dT/dz , similar to Taylor expansions.

The temperature distribution is converted to the variables $\vartheta(i)$, assembled in the vector $\boldsymbol{\vartheta}$.

With the surface temperature TsK and the height z(i) we get expressions for TLR(i) and $\vartheta(i)$:

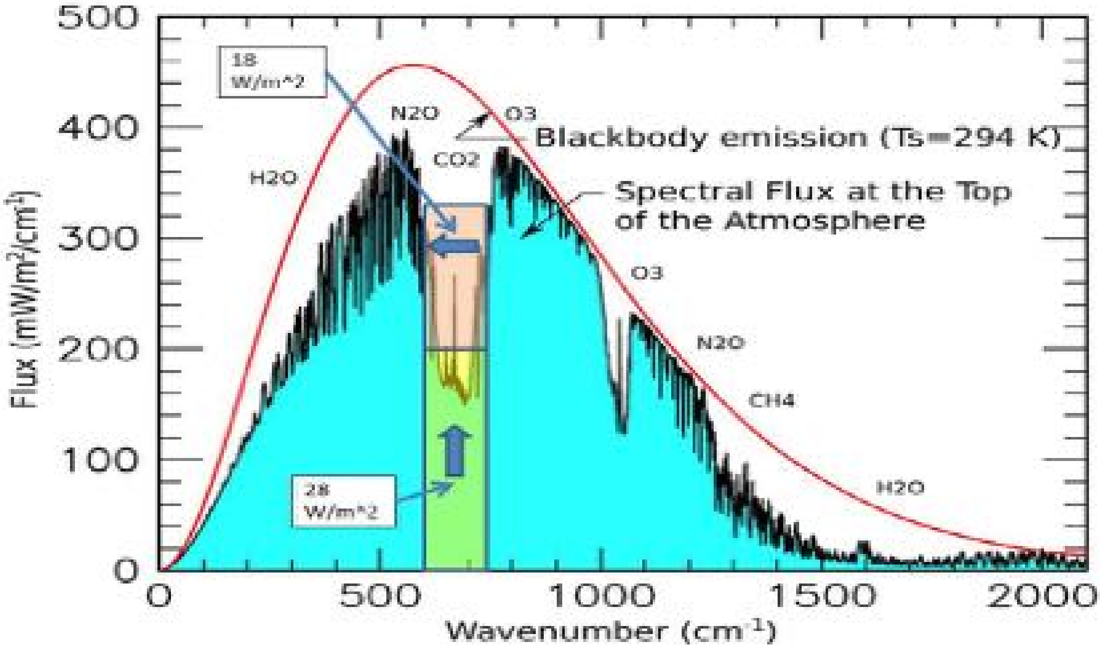
$$TLR(i)=TsK + L*z(i) \qquad \text{and} \qquad \vartheta(i) = \sigma*(TsK + L*z(i))^4 \qquad \textbf{(5)}$$

Where z(i) < 11.5 is the vertical coordinate of the grid in km.

For z(i) >11.5 km – for the CO_2 analyses -– the temperature distribution follows from **figure 4**, which corresponds to the Standard Atmosphere. Outer-space temperature is taken as zeroK = 0 or $\vartheta(N) = 0$.
In **figure 4** are also depicted the *normalized* distribution of water vapor and of carbon dioxide gas: fdH2O and fdCO2, respectively.
The *normalized* H_2O distribution is defined heuristically by an exponential drop : fdH2O(z) = exp(-mH2O*z/height5)
The coefficient mH2O = 7, for a reference height5 of 5 km, is obtained by comparing the results with main-stream K&T papers on the subject.
The fdCO2 distribution is defined by the pressure and temperature dependent air density, assuming a constant *volumetric* concentration of CO_2 in ppm over the height. See **Appendix 2**

Fractions of H2O and CO2 in the LW terrestrial spectrum

From **figure 5** we can conclude that the fraction of CO2 in the spectrum is 28 +18 =46 W/m^2 of the total Prevost flux = σ*TsK^4 = 394 W/m^2 for a surface temperature of TsK = 288.72:
fractionCO2 = 0.1168 and fraction H2O= 1-fractionCO2 = 0.8832
Figure 5 from Pangburn blog **[2]**



Thermal radiation from below assessed from top-of-atmosphere.
Original graph from NASA

NB Ts =294 K in **figure 5** is a reference value for the red Planck curve.
Other data are for a temperature of TsK = 288.72.

Results of the stack model for water-vapor.

From **figure 4** we see, for the evacuation of heat through an atmosphere with only water-vapor, a model with a height of 11.5 km is sufficient.

The computer program (listing in **Appendix 3**) includes a mesh generator with element sizes based on geometric series: for N=50 nodes of order of 0.1 meter at the surface and of 1.75 km at 11.5 km heigh.

Figure 6 gives a graphical display of the vector relation **(4): q = K*ϑ.**

It might be useful to repeat in words what the vector relation means:

for a measured temperature distribution given in 50 nodes by a vector of parameters **ϑ** $[W/m^2]$ *of order 50 and by multiplication with a dimensionless radiation matrix* ***K*** *for water-vapor of order 50x50, one obtains a vector* ***q*** $[W/m^2]$ *of order 50.*

What is the physical interpretation of components of vector **q** $\mathbf{[W/m^2]}$?

They represent: q(1) = qsurfH2O, LW surface flux into H_2O vapor column
-q(50) = OLRH2O, outgoing LW radiation from H_2O vapor.

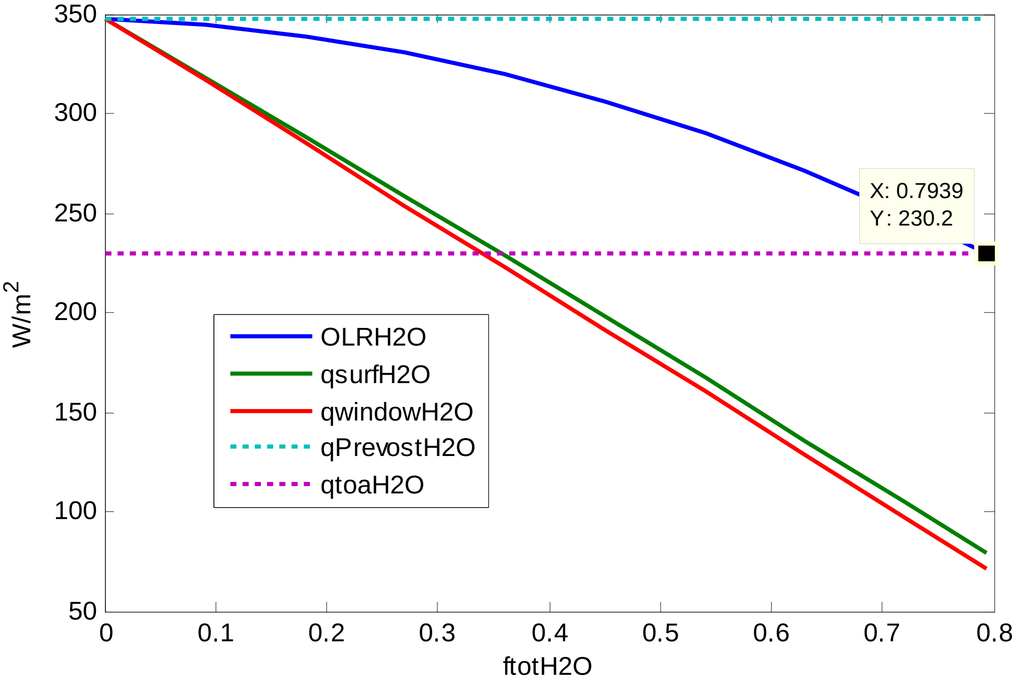
We see in **figure 6** these two components of the vector **q** as function of ftotH2O as well as qtoaH2O =230.12 , qPrevostH2O and qwindowH2O:

$$qPrevostH2O = fraction_{H2O} * \sigma * TsK^4 = fraction_{H2O} * \vartheta(1)$$

$$qwindowH2O = (1 - ftot_{H2O}) * qPrevost_{H2O}$$

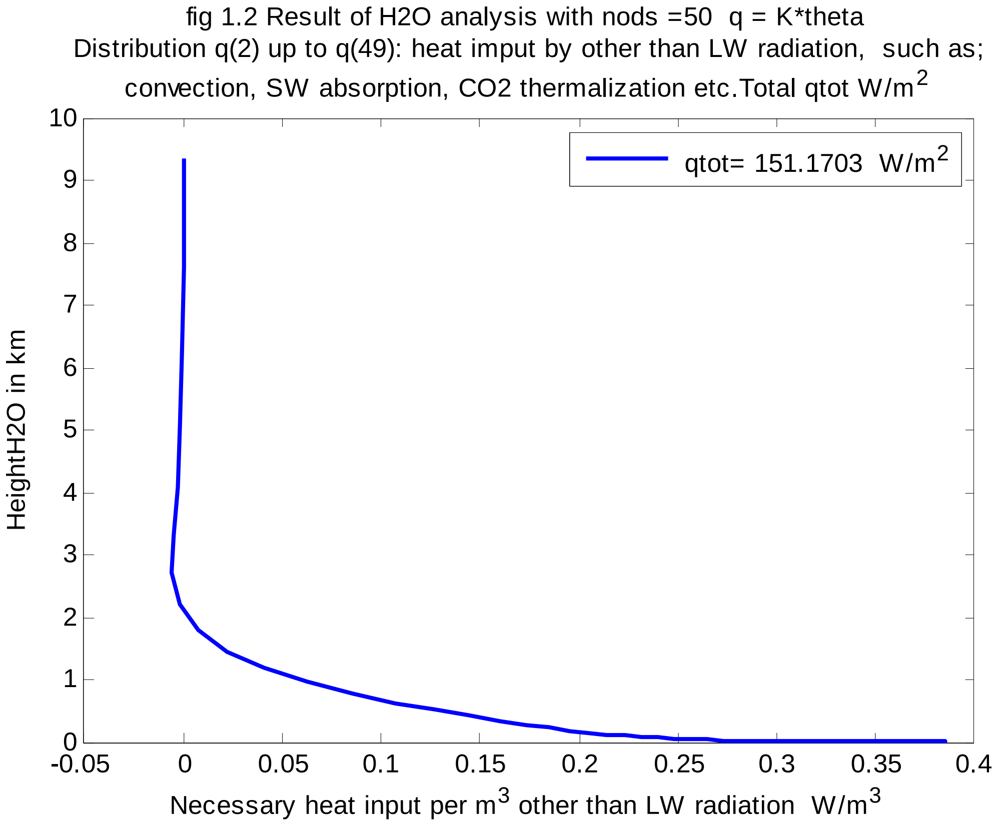
Figure 6

fig 1.1 Result of H2O analysis with nods =50
nods = 50 mH2O = 7 heightH2Okm = 11.5 LdT/dz = -6.5 degree/km TsK = 288.7209
ftotH2O = 0.7939 fractionH2O = 0.88325 qtoaH2O = 230.176 qPrevostH2O = 347.9998
OLRH2O, qsurfH2O as function of ftotH2O. qwindowH2O = (1-ftotH2O)*qPrevostH2O



For ftotH2O = 0.7939 a value OLR_{H2O} = 230.18 W/m²: it is the average of the global outgoing LW radiation at H_2O frequencies, for which the stack model gives ftotH2O= 0.7939 and a window of (1-ftotH2O) = 0.2061. *With OLR_{CO2} = 9.82 W/m² from* ***figure 8****, the average total outgoing heat flux at the top of atmosphere is: OLR_{H2O} + OLR_{CO2} = qtoa = 240 W/m².* The calculated values of the other components of the vector **q** for i = 2 to 49 are given in **figure 7** as function of the height, not as nodal values with dimension W/m² but as distribution in W/m³.

Figure 7



The stack model shows, apart from LW radiation from the surface of the planet , the additional heat input distribution from the atmosphere with a total value of: 151.17 W/m².

Possible atmospheric heat inputs are from:

- *absorption of incoming SW radiation by aerosols*
- *convection from the surface of sensible and latent heat,* and
- *thermalization of CO_2 i.e. absorbance in the atmosphere of a part of the infra-red-active gas by LW radiation from the surface but not re-emitted. The heat is given by collisions to the bulk of the atmospheric molecules: 78% N_2, 21 % O_2 and the trace water-vapor gas H_2O.*

The first two contributions are also mentioned by main-stream authors in so-called K&T global energy budgets. The third possible contribution, the thermalization of CO_2 introduced in AD 2016 by Pangburn [2], seems to be ignored by many authors on the subject.
We come back on the phenomenon thermalization further on.

Dependence of OLR_{H2O} on surface temperature

For studies related to the dependence on the ambient temperature of the evacuation of heat from the planet by LW radiation, we need the variation of OLR_{H2O} with the surface temperature TsK.

We repeat equation **(5)** giving the relation between TLR(i) en ϑ(i):

$$TLR(i)=TsK + L*z(i) \quad \text{and} \quad \vartheta(i) = \sigma*(TsK +L*z(i))^4 \qquad \mathbf{(5)}$$

The variation of ϑ(i) with the surface temperature is defined by the derivative. For constant lapse rate L:

$$d\vartheta(i)/dTsK = 4\ \sigma*(TsK +L*z(i))\text{^}3 = 4*\vartheta(i)\ /TLR(i)$$

We define the vector **dϑdTsK** consisting of 50 components dϑ(i)/dTsK.

The derivative of the vector **q** with TsK becomes the vector:

$$\mathbf{dqdTsk} = \mathbf{K}*\mathbf{d\vartheta dTsK}$$

The derivative of OLR_{H2O} with TsK becomes the minus 50th component:

$$dOLR_{H2O}/dTsK = -\ dqdTsK(50)$$

The stack calculations give: $dOLR_{H2O}/dTsK= 3.2342$ [W/m²/ºK]
We find a relation for the increase of OLR_{H2O} due to the surface temperature increase. We use the IPCC name for it, $forcing_{OLR}$:

$$forcing_{OLR} = (dOLR_{H2O}/dTsK)*deltaTsK \qquad \mathbf{(6)}$$

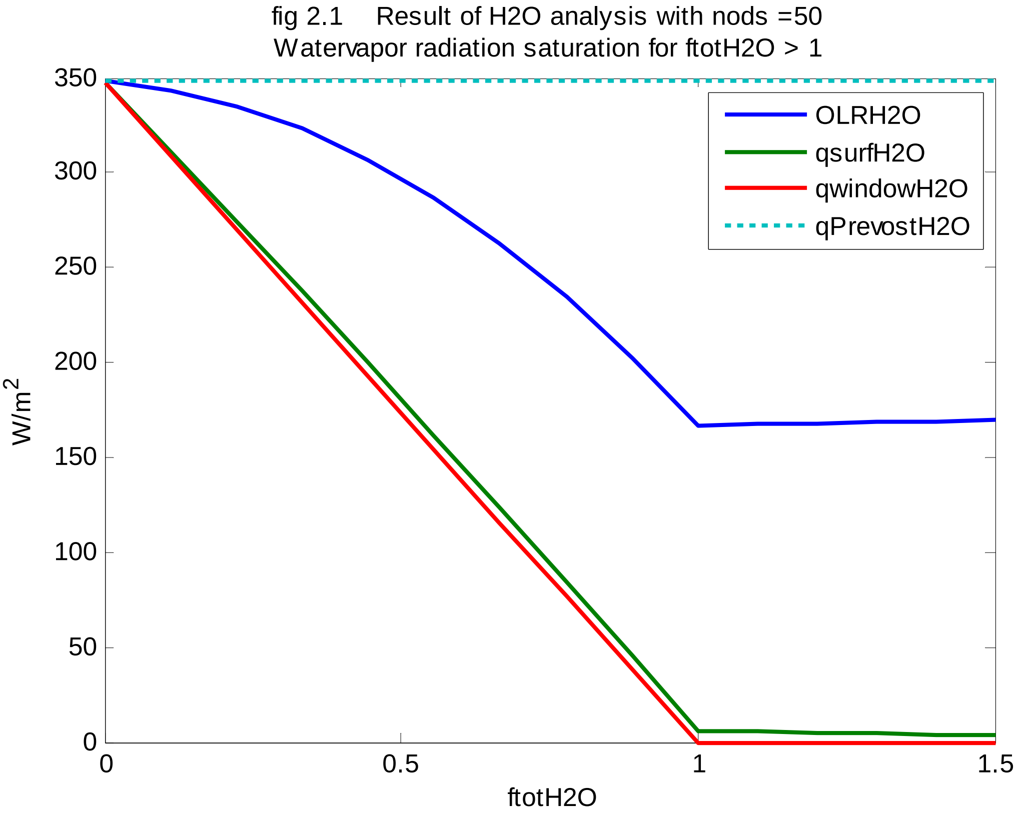
Saturation of dense infra-red-active gases

In **figure 8** are given the results of analyses for water-vapor concentrations with ftotH2O >1. We see that the OLR_{H2O} is not decreasing any more. The phenomenon is called “saturation” and is explained by equation **(3b)**, repeated here:

$$viewfactor(i,j) = 1 - \sum f(k) \quad \text{for} \quad z(i)<z(k)<z(j) \tag{3b}$$

For $\sum f(k) > 1$, viewfactor(i,j) would become negative: **it is put to zero.** For viewfactor(i,j) = 0 there is no LW radiation between the node i and the node j of the element(i,j).

Figure 8



The saturation phenomenon does not appear for water-vapor in the atmosphere because the concentration remains low with ftotH2O < 1. It is shown here for water-vapor — *for demonstration purposes only* — because it is important for the CO2 analyses further on, with ftotCO2 >1. Authors very often ignore the CO2 saturation phenomenon, although it is one of the two reasons — *saturation and thermalization of CO2* — for the planet not heating up, as will be shown in the next sections.

Results of the stack model for carbon dioxide gas CO2

The stack model for H_2O vapor is a simple one-stream, mono-chromatic model of the evacuation of heat from a planet with only H_2O vapor without CO_2 gas. It turned out to be accurate enough when compared to the results of main-stream authors on the subject in so-called global K&T diagrams adjusted for the non-physical back-radiation in the two-stream models. It can also be used for the analysis of carbon dioxide gas with saturation for values of ftotCO2>1.

For the CO_2 analysis we take a model with a height of 30 km with the three 3 temperature zones according to **figure 4 ,** and 100 nodes with element sizes of 1.4 m at the surface varying to 700 m at a height of 30 km, defined by geometric series.

The results of the vector relation **(3)** for CO_2, **q = K*ϑ,** are given in **figure 9**, which is equivalent to **figure 6** for the H_2O vapor analysis.

The components of **q ,** qPrevostCO2 and qwindowCO2 represent now:

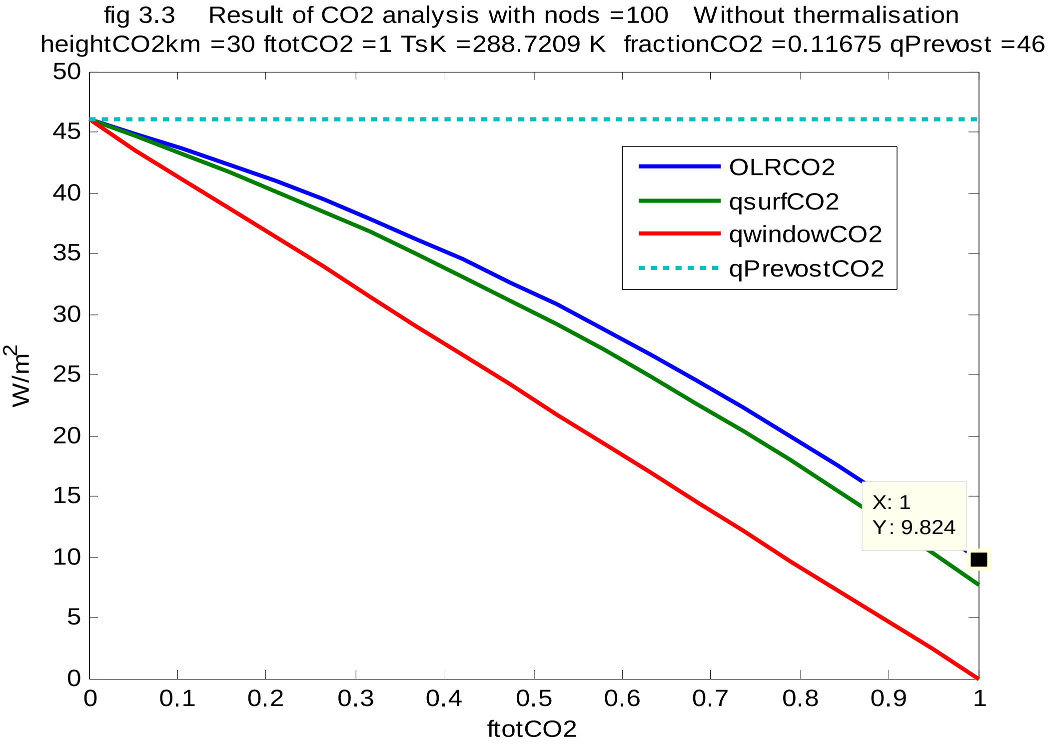
$$q(1) = q_{surf\,CO2} = \text{LW surface flux into the } CO_2 \text{ column}$$

$$-\,q(100) = OLR_{CO2} = \text{outgoing LW } CO_2 \text{ radiation}$$

$$qPrevost_{CO2} = fraction_{CO2} * \sigma * TsK^4 = fraction_{CO2} * \vartheta(1).$$

$$qwindow_{CO2} = (1 - ftot_{CO2}) * qPrevostCO2$$

Figure 9 *no thermalization and no saturation*



Surface temperature increase due to CO$_2$ gas increase.

In **figure 9** we see a decreasing OLR_{CO2} flux, from qPrevostCO2 = 46 W/m² for ftotCO2 = 0 towards a value of 9.824 W/m² for ftotCO2 = 1. The decrease $deltaOLR_{CO2}$ as function of OLR_{CO2} becomes:

$$deltaOLR_{CO2} = -(qPrevostCO2 - OLR_{CO2}) \qquad \textbf{(7)}$$

In order to keep the sum $OLR_{H2O} + OLR_{CO2}$ = qtoa = 240 W/m² constant, the necessary increase of OLR_{H2O} due to the increase of the surface temperature TsK, called forcingOLR by IPCC, is the opposite:

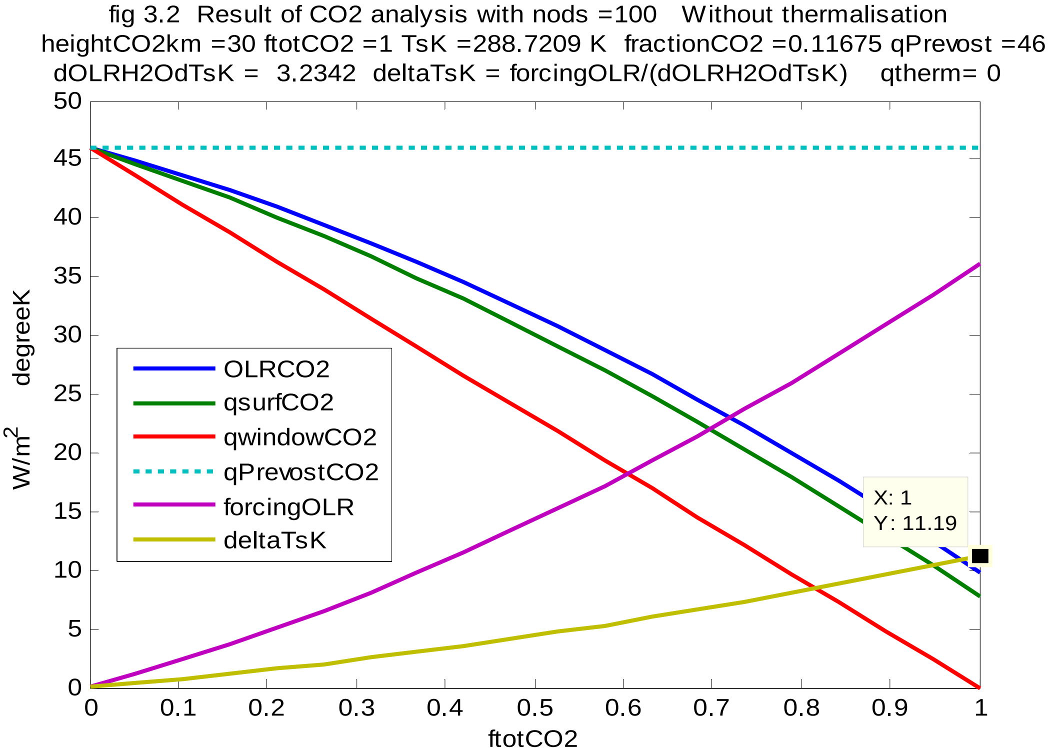
$$forcingOLR = - deltaOLR_{CO2} = (qPrevostCO2 - OLR_{CO2}) \qquad \textbf{(7a)}$$

With the corresponding surface temperature TsK increase from equation **(6)** repeated here:

$$deltaTsK = forcingOLR/(dOLR_{H2O}/dTsK) \qquad \textbf{(6)}$$

In **figure 10** are given the results from equations **(6)** and **(7 a)** for CO$_2$ concentrations from ftotCO2 = 0 to 1, or 400 ppm in a 30km high column.

Figure 10 *no thermalization and no saturation*



Historical omissions by IPCC authors

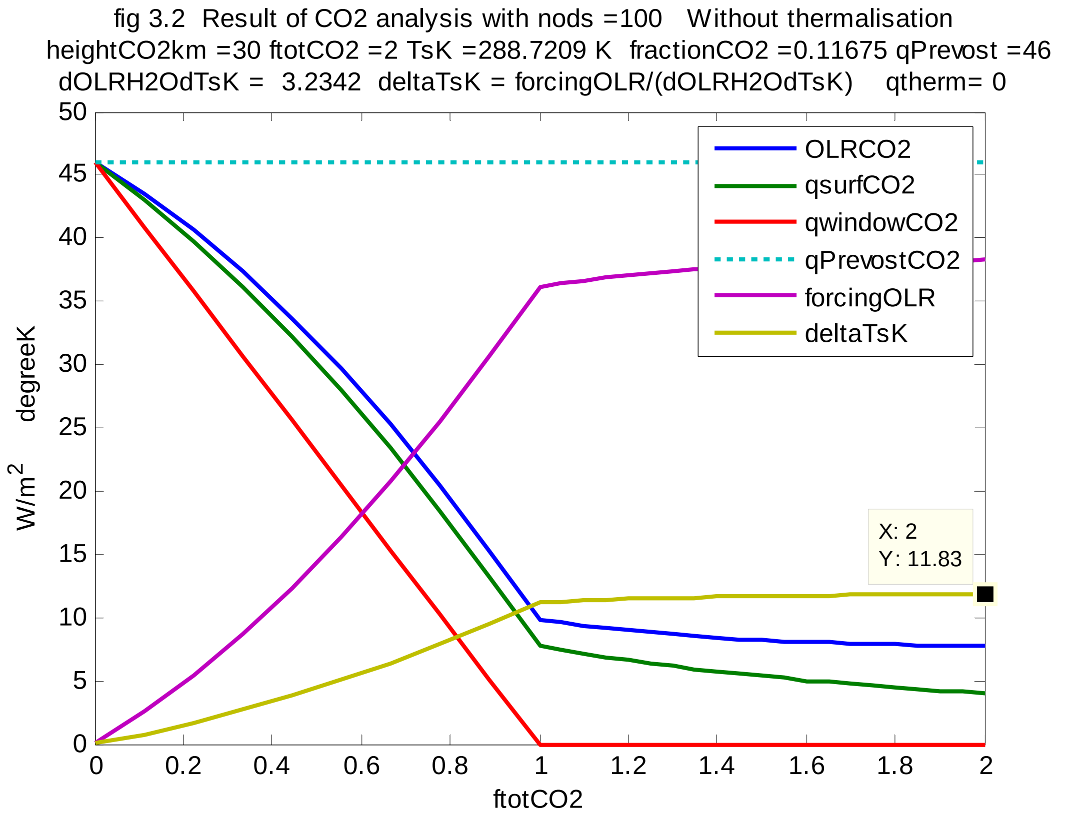
In this paper the simple stack models are used to analyze the hindering of LW radiation by infra-red-active gases like water vapor H_2O and carbon dioxide CO_2, respectively in **figure 6** and **9.**
In **figure 10** are given the results from equations **(5)** and **(6a)** for CO_2 concentration increasing from ftotCO2 = 0 to 1, or 400 ppm for an atmospheric column of 30 km high.
With the two-stream Schwarzschild procedures of AD 1906 one can obtain similar numbers but they are seldom shown in detail by IPCC authors.
It is the advantage of the one-stream stack model with efficient graphical display that results can be shown, and not only for one single CO_2 value.
The average slope of the deltaTsK curve in **figure 10** between values of ftotCO2 = 0 and ftotCO2 =1 becomes 11.19°C.
The so-called sensitivity analysis for double CO_2 concentration, for the theory behind **figure 10**, could be represented by this slope.

It were this kind of temperature increases that James Hanssen was referring to in the congressional hearing of 23 June 1988, organized by then senator Al Gore in the Washington DC Rayburn building with a defected air-condition.
It was difficult to look into the future by extrapolation.
However the temperature of the pre-industrial period at 280 ppm was known by measurement and smaller than the corresponding value from **figure 10**: for ftotCO2 = 0.7 = 280 ppm, the value for deltaTsK = 6.7 °C.
From the very beginning skeptics have argued that when the theory underlying **figure 10** is not accurate to explain the past, one cannot claim an extrapolation towards higher CO_2 concentrations.
Obviously the IPCC slogan ''*science is settled*'' is wrong.
Nevertheless, a value of deltaTsK around 11°C for ftotCO2 = 1 remained to be the temperature from the Schwarzschild analyses, still in use by alarmists and even by some skeptics as starting point for future temperature analyses.
Already in AD 2016 , Pangburn **[2]** has given the reason for the too high numbers in **figure 10**: thermalization of CO_2.
We come back to the thermalization phenomenon further on.

Saturation of carbon dioxide

So-called “saturation of CO_2” is another phenomenon often ignored by IPCC authors with the excuse of the slogan " *science is settled* ". Saturation results for a 30km high CO_2 column for concentrations beyond ftotCO2=1, or 400 ppm, are shown in figure **figure 10a.**

Figure 10a *with saturation only, no thermaliztion*



The temperature increase from ftotCO2=1 up to ftotCO2 =2 is small:

$$11.83 - 11.19 = 0.64\ °C$$

These temperature increases beyond ftotCO2 = 1 are even smaller than the Happer numbers for the region between ftotCO2 = 1 and = 2 given in [**3**]**.** We note that for ftotCO2 = 1 the deltaTsK value around 11 °C is too high because thermalization has not been taken into account.

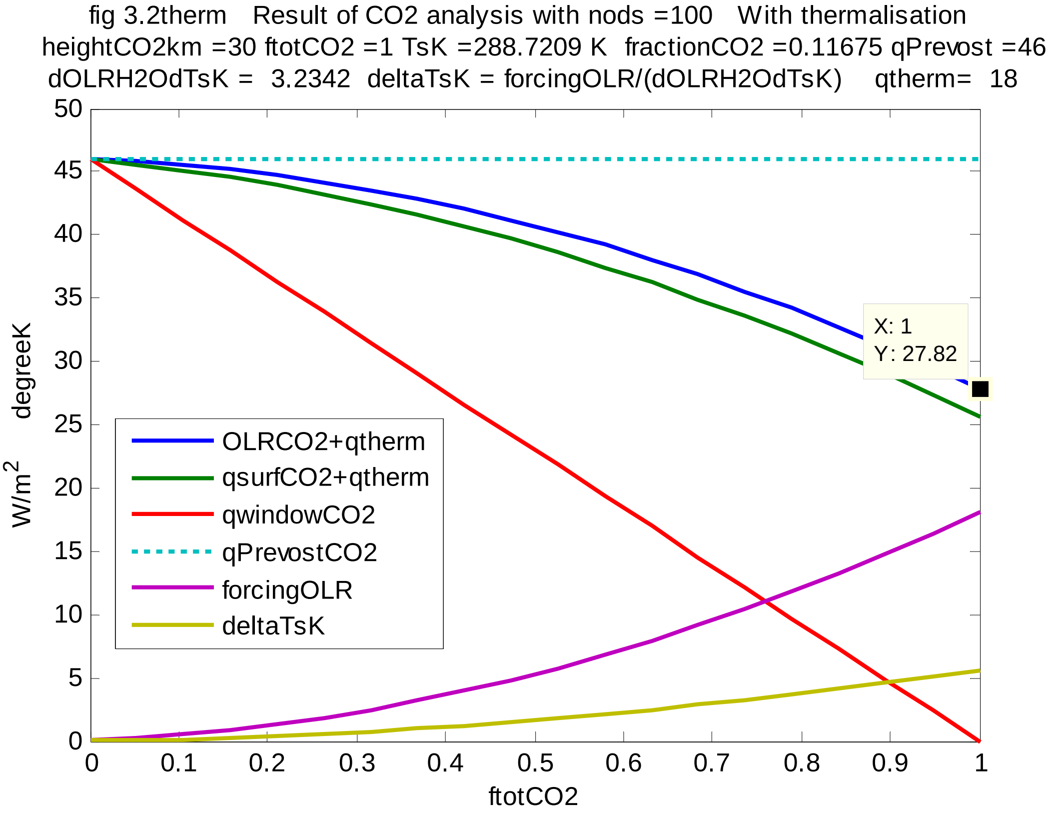
Thermalization of CO_2.

The classical Stefan-Boltzmann-Christiansen relation assumes that the information exchange concerning the temperatures between surfaces and thereby exchange of energy is immediate.
There are however different time delays in the process, according to Pangburn **[2].**
Relaxation time:
The time it takes for absorbed energy to be shared with surrounding molecules by means of molecular collisions, known as diffusion.
It is of the order of a few microseconds.
Decay time:
The time between the absorption process of a molecule CO_2 and the re-emission process, it averages about 1.1 second.

Since the relaxation time is much smaller than the decay time, a CO_2 molecule in the absorption phase collides many times with surrounding 78% nitrogen N_2 molecules and 21% oxygen O_2 molecules, as well as with infra-red-active water-vapor H_2O molecules.
The CO_2 molecule loses the surplus energy before it has been built up completely for re-emission!
The CO_2 molecule is said to be thermalized, the surplus energy goes to the bulk of the molecules of the atmosphere.
The exchanged energy has lost its CO_2 identity, the broad band of H_2O frequencies is used for LW radiation to outer space according to **figure 7:** we see that H_2O molecules pick up 151.17 W/m² from the atmospheric sources other than LW radiation from the surface.
See **figure 4** and Pangburn blog **[2]** for further details

In the stack model we have taken into account the thermalization results of Pangburn: for 400 ppm, a fraction 18 W/m² is thermalized and is radiated to outer space by means of the broad water vapor H_2O frequency bands according to **figure 7**.
Figure 11 gives results with a thermalization of qtherm = 18 W/m², to be compared with **figure 10,** without thermalization.
For values ftotCO2 = 0 to 1 the distribution of qtherm is assumed to be linear between 0 and 18 W/m² in **figure 11**.

Figure 11 *with thermalization, saturation not applicable for ftotCO2<1*



For ftotCO2 = 1:

q(1) = qsurf$_{CO2}$ = LW surface flux into CO_2 column,
including 18 W/m² to be thermalized
and re-radiated by H_2O molecules

- q(100) = OLR$_{CO2}$ = outgoing LW 9.82 CO_2 path + 18 H_2O path,
total 27.82 W/m²

Pangburn results **[2]** with qtherm = 18 W/m² are given for ftotCO2=1, or 400 ppm in a stack of 30km high in the gravitation field.
In **figure 11** the thermalization contribution is assumed to be linear between zero for ftotCO2 = 0 and 18 W/m² for ftotCO2=1.
For the pre-industrial value of ftotCO2 = 0.7 or 280 ppm the value deltaTsK becomes about 3.00 and for ftotCO2 =1 the value of deltaTsK = 5.603: a difference of 2.6 degree. This difference is included in **Table1**.

In **figure 11** for ftotCO2<1, only the thermalization is taking into account. For CO2 concentrations beyond the 400 ppm of the year 2015 AD, there is not only thermalization but also saturation, as shown in **figure 12** with the final result of this paper.

Figure 12 *with both thermalization and saturation*

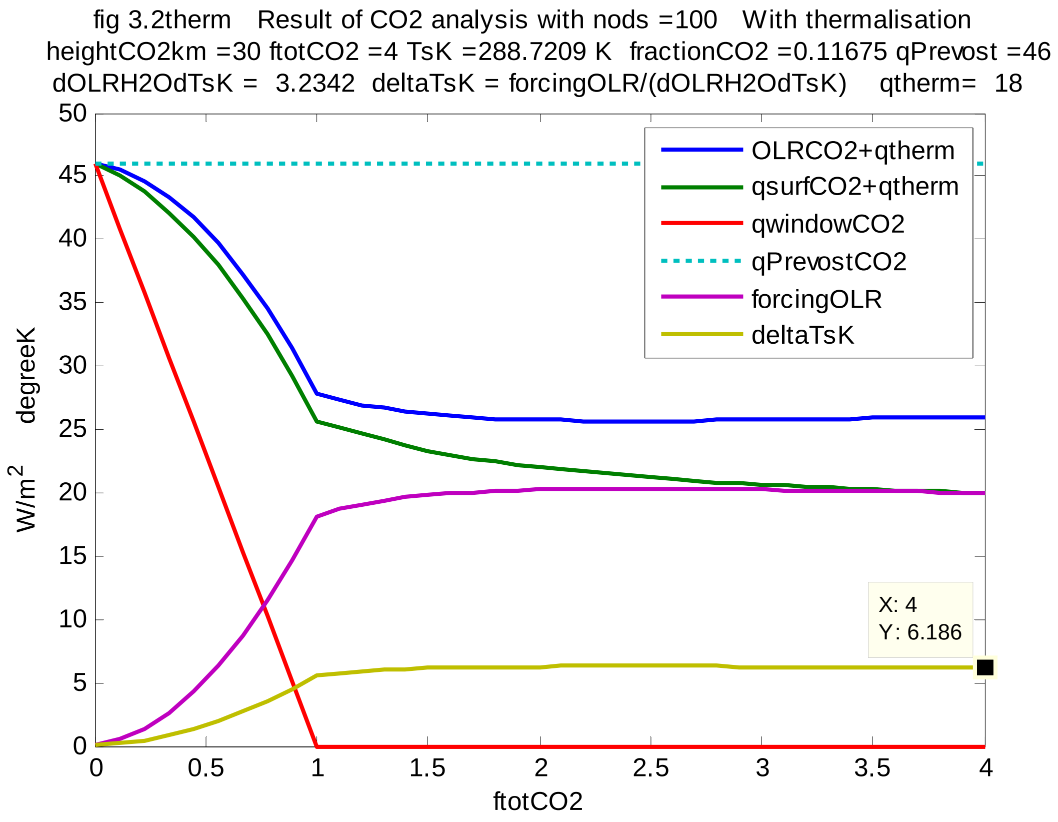


Table 1 summarizes the result of **figure 11** and **figure 12**, giving the temperature increase from ftotCO2 = 0 to ftotCO2 = 4, or 1600 ppm around the year AD 2615, assuming an increase of 2 ppm/yr.

Table 1

Temperature variations in degree C from fotCO2 = 1.

due to thermalization (qtherm = 18 W/m²) and saturation

ftotCO2	ppm	deltaTsK	deltaTsK -5.62	year AD
0.7	280	3.00	**-2.62**	
1	400	5.62	**0**	2015
2	800	6.265	**0.645**	~ 2215
4	1600	6.186	**0.566**	~ 2615

The difference of - 2.62 °C between the pre-industrial values of 280 ppm or ftot = 0.7 depends on the distribution of the thermalization for ftotCO2<1.

The indicated temperature variations are only due to CO2 concentration variations and for a thermalization of 18 W/m2, reported by Pangburn for CO2 concentration of 400 ppm.

Constant Sun intensity and constant cloud influences are assumed.

Conclusions

The one-stream stack model for infra-red-active trace gases, based on the classical one-stream Stefan-Boltzmann-Christiansen relation, already validated for the analysis of LW radiation through an atmosphere with H_2O vapor in so-called K&T energy budgets, has now also been applied for the analysis of CO_2 gas and thereby increases of the surface temperature of the planet. The MATLAB.m files are given in **Appendix 3**

The stack model is a simple model.
It deals with the issues of thermalization and saturation of CO_2 in a transparent way.
The two phenomena, thermalization and saturation of CO_2 in a 30km high column of , give rise to small temperature increases, while

- before the year AD 2015 with CO_2 concentration below 400 ppm the thermalization of CO_2 was not published yet by Pangburn

- since the year AD 2015, with higher than 400 ppm CO_2, both thermalization **and** saturation of CO_2 are ignored by IPCC.

Thermalization of CO_2 as reported by Pangburn **[2],** give even lower temperature increases as compared to the saturation only analyses of Happer **[3].**

The infra-red-active gas CO_2 is harmless and non-polluting.

Until nuclear power has been installed back again, we need fossil fuels to generate electricity and for combustion engines of cars, boats and planes.

DRILL BABY, DRILL

Burning fossil fuels — **fortunately** — will increase the concentration of CO_2, also called "greenhouse" gas, which indeed is used in nursery greenhouses to boost the growth of vegetables and flowers.

CO_2 is food for plants

We need more atmospheric CO_2 to feed the growing world population.

Acknowledgment

The author wants to thank Claes Johnson **[4]** who inspired him to write this paper based on the one-stream LW radiation to outer space, avoiding the nonphysical back-radiation of the two-stream approach of Schwarzchild of 1906.
The author interpreted the one-stream proposals from Johnson by using the Stefan-Boltzmann-Chrisiansen relation always for a pair of surfaces, enabling the concept of standing waves between resonating infra-red-active molecules with the same eigen-frequency.

Thanks to Dan Pangburn **[2]** for his authorization to include **figure 4** in this paper, and his explanations about thermalization.

Thanks to John O'Sullivan at Principia Scientific International for the publication of the more condensed version of this paper.**[1]**

References

[1]Reynen
https://principia-scientific.com/wp-content/uploads/2023/04/SaturationIVnew.pdf

https://principia-scientific.com/wp-content/uploads/2025/01/Stackmodel20jan2025.pdf

[2]Pangburn**,**
https://www.researchgate.net/publication/316885439_Climate_Change_Drivers.
[3]Happer,
https://www.youtube.com/watch?v=CA1zUW4uOSw
[4]Johnson,
https://computationalblackbody.wordpress.com/

Appendix 1

Stefan-Boltzmann-Christiansen Relation for energy radiation

We consider the energy radiation of surfaces and use the symbols:

- ε coefficient for emission,
  representing the fraction of emitted energy for non-black surfaces compared to the emitted energy of black surfaces. For black surfaces $\varepsilon = 1$, and the energy emitted by a black surface is
  $q = \sigma_b T^4$ (Stefan-Boltzmann relation, $\sigma_b = 5.67$ e-8 W/m²/ °K⁴)
- α coefficient for absorption,
  representing the fraction of energy absorbed as compared to total energy arriving at the surface.
- ρ coefficient for reflection,
  representing the fraction of energy reflected as compared to total energy arriving at the surface.
- τ coefficient for transparency,
  representing the fraction of energy transmitted through the body as compared to the total energy arriving at the surface.

From the definitions of α , ρ and τ it follows:

$$\alpha + \tau + \rho = 1 \qquad \textbf{(A1.1)}$$

In **figure A1.1** are shown plates 1 and 2 by two vertical lines with two horizontal arrows towards each other, representing energy fluxes:

Figure A1.1

The arrow out of plate 1 represents q_1 .
It is ıthe total energy in W/m² from plate 1 to plate 2, consisting of the emitted flux $\sigma_1 T_1^4$ out of plate 1, plus the reflection $\rho_1 q_2$ by plate 1 of the flux q_2 from plate 2.
The parameters of the arrow out of plate 2 are obtained by cyclic permutation of the subscripts.

We get two simultaneous equations for q_1 and q_2:

$$\begin{aligned} q_1 &= \sigma_1 T_1^4 + \rho_1 q_2 \\ q_2 &= \sigma_2 T_2^4 + \rho_2 q_1 \end{aligned} \qquad \textbf{(A1.2)}$$

In matrix notation:

$$\begin{vmatrix} 1 & -\rho_1 \\ -\rho_2 & 1 \end{vmatrix} \begin{vmatrix} q_1 \\ q_2 \end{vmatrix} = \begin{vmatrix} \sigma_1 T_1^4 \\ \sigma_2 T_2^4 \end{vmatrix} \qquad \textbf{(A1.2a)}$$

By inverting the matrix we find, with determinant det = $1 - \rho_1 \rho_2$:

$$\begin{vmatrix} q_1 \\ q_2 \end{vmatrix} = 1/\mathrm{det} \begin{vmatrix} 1 & \rho_1 \\ \rho_2 & 1 \end{vmatrix} \begin{vmatrix} \sigma_1 T_1^4 \\ \sigma_2 T_2^4 \end{vmatrix} \qquad \textbf{(A1.3)}$$

The algebraic expressions for q_1 and q_2 become:

$$\begin{aligned} q_1 &= (\sigma_1 T_1^4 + \rho_1 \sigma_2 T_2^4)/\mathrm{det} \\ q_2 &= (\sigma_2 T_2^4 + \rho_2 \sigma_1 T_1^4)/\mathrm{det} \end{aligned} \qquad \textbf{(A1.3a)}$$

Kirchhoff and Christiansen relations for non-transparent plates

For $\tau = \tau_1 = \tau_2 = 0$ the relation **(A1.1)** becomes:

$$\rho = 1 - \alpha \qquad \textbf{(A1.1a)}$$

With $\rho_1 = 1 - \alpha_1$, $\rho_2 = 1 - \alpha_2$, $\sigma_1 = \varepsilon_1 \sigma_b$, $\sigma_2 = \varepsilon_2 \sigma_b$ we find

$$q_{1,2} = q_1 - q_2 = (\alpha_2 \varepsilon_1 \sigma_b T_1^4 - \alpha_1 \varepsilon_2 \sigma_b T_2^4)/\mathrm{det} \qquad \textbf{(A1.4)}$$

When the plates have the same temperature, — $T_1 = T_2$ —, there is no heat flux between the plates and we find the relation:

$$\alpha_2 \varepsilon_1 - \alpha_1 \varepsilon_2 = 0 \qquad \text{or} \qquad \varepsilon_1/\alpha_1 = \varepsilon_2/\alpha_2$$

By a thought process with e.g. plate 1 is black with $\varepsilon_1 = \alpha_1 = 1$ we find that

$$\varepsilon_2/\alpha_2 = 1 \qquad \text{or} \qquad \varepsilon_2 = \alpha_2 .$$

Again by a thought process, the value of the emission coefficient of the surface of plate 2 does not depend on the surface of plate1, and the conclusion is that emission and absorption coefficients are always equal:

$$\varepsilon = \alpha \qquad \textbf{(A1.5)}$$

This is the Kirchhoff relation of 1860.

From equation **(A1.5)** we find :

$$q_{1,2} = \alpha_1 \alpha_2 \sigma_b (T_1^4 - T_2^4)/\det \qquad \textbf{(A1.4a)}$$

The determinant, det = 1- $\rho_1 \rho_2$, becomes with the relation **(A1.1a)**:

$$\det = 1-(1-\alpha_1)(1-\alpha_2) = \alpha_1 + \alpha_2 - \alpha_1 \alpha_2 \qquad \textbf{(A1.6)}$$

We find: $\alpha_{12} = \alpha_1 \alpha_2 / (\alpha_1 + \alpha_2 - \alpha_1 \alpha_2)$ or $1/\alpha_{12} = 1/\alpha_1 + 1/\alpha_2 - 1$

Expressed in emission coefficients:

$$q_{1,2} = \sigma_{1,2} (T_1^4 - T_2^4) \qquad \textbf{(A1.7)}$$

with $1/\sigma_{1,2} = 1/\sigma_1 + 1/\sigma_2 - 1/\sigma_b$ $\quad \sigma_{1,2} = \varepsilon_{1,2} \sigma_b$
$1/\varepsilon_{1,2} = 1/\varepsilon_1 + 1/\varepsilon_2 - 1$

This is the Christiansen relation from 1883.

When both surfaces have emission coefficients = 1 then also $\varepsilon_{1,2} = 1$.
When only one surface 1 has an emission coefficient =1 then $\varepsilon_{1,2} = \varepsilon_2$.
When only one surface 2 has an emission coefficient =1 then $\varepsilon_{1,2} = \varepsilon_1$..
When the emission coefficients of both plates are $\varepsilon < 1$ then $\varepsilon_{1,2} = \varepsilon/(2 - \varepsilon)$.

The Kirchhoff and Christiansen relations **(A1.5)** and **(A1.7)** are valid for non-transparent plates ($\tau_1 = \tau_2 = 0$) and are radiation frequency dependent.

The Christiansen relation for transparent plates.

The grids in the stack model are transparent.
Relation **(A1.1a)** is not valid anymore, we have to go back to relation **(A1.1)**:

$$\alpha + \tau + \rho = 1 \qquad \textbf{(A1.1)}$$

We can introduce parameters $\beta = \alpha + \tau$ and equation **(1)** becomes:

$$\beta + \rho = 1 \qquad \text{and} \qquad \rho = 1 - \beta = 1 - (\alpha + \tau) \qquad \textbf{(A1.1b)}$$

Inserting these reflection coefficients in the relations **(A1.4)** it is sufficient to replace the α by β and we get:

$$q_{1,2} = q_1 - q_2 = (\beta_2 \varepsilon_1 \sigma_b T_1^4 - \beta_1 \varepsilon_2 \sigma_b T_2^4)/(\beta_1 + \beta_2 - \beta_1 \beta_2) \qquad \textbf{(A1.4b}$$

For the grids in the stack model the transmission coefficients τ are nearly equal 1 and we will use the f coefficients:

$$\tau = 1 - f \qquad \beta = \alpha + 1 - f = 1 - (f - \alpha) \qquad \rho = 1 - \beta = f - \alpha$$

The transmission through the elements is taken care of by the viewfactors. The grids of infra-red-active molecules in the stack model of the main text can be considered, in steady-state conditions, as grids with $\alpha = 0$.
From the 3 parameters of (1) only two remain: τ and ρ:

$$\tau + \rho = 1 \qquad \textbf{(A1.1c)}$$
$$\tau = 1 - f \qquad \rho = f \qquad \beta = f$$

With relations (A1.1c) we obtain:

$$q_{1,2} = q_1 - q_2 = f_{1,2}(\sigma_b T_1^4 - \sigma_b T_2^4) \qquad \textbf{(A1.4c})$$

$$f_{1,2} = f_1 f_2 / (f_1 + f_2 - f_1 f_2)$$

The Stefan-Boltzmann-Christiansen for radiation between the grids in the stack model become like equation **(A1.7)**

$q_{1,2} = \sigma_{1,2} (T_1^4 - T_2^4)$ with $1/\sigma_{1,2} = 1/\sigma_1 + f_1/f_2 (1/\sigma_2 - 1/\sigma_b)$ **(A1.7a)**

The relation A1.7a is given by Christiansen for radiation between a smaller convex body inside a greater concave body.
C. Christiansen, " Annalen der Physik und Chemie" , Leipzig, 1883.
In the stack model, we can treat the f coefficients, representing relative surfaces in the same way as emission coefficients with the relation:

$$1/f_{1,2} = 1/f_1 + 1/f_2 - 1 \qquad \textbf{(A1.7b)}$$

When both factors f_1 and f_2 in an element are 1 — in case of a radiation between the surface and outer-space — the coefficient $f_{1,2}$ becomes also 1 although the heat flux is decreased by viewfactor(1,nods) = 1-ftot, and viewfactor(1,nods) = 0 for ftot>1.
When only one surface 1 has an emission coefficient $f_1 = 1$ then $f_{12} = f_2$
When only one surface 2 has an emission coefficient $f_2 = 1$ then $f_{12} = f_1$.
When the two emission coefficients both are $f < 1$ then $f_{12} = f/(2 - f)$

For the CO2 analyses the coefficients for ftot =1 or 400 ppm become of the order of 10e-4 and the CO2 analysis can be reduced by one single element from surface to outer-space at 30 km high ,with the reduction by means of the viewfactors.

For the water vapor analysis however, the stack model has shown that the water vapor picks up about 150 W/m² from the atmosphere over a height of only 2 km.
For the water vapor analyses the Christiansen relation is necessary.

Appendix 2

Physical properties of the atmosphere

In **figure A2.1** are given the data, needed for the analysis of the LW heat transport in the atmosphere.

FigureA2.1

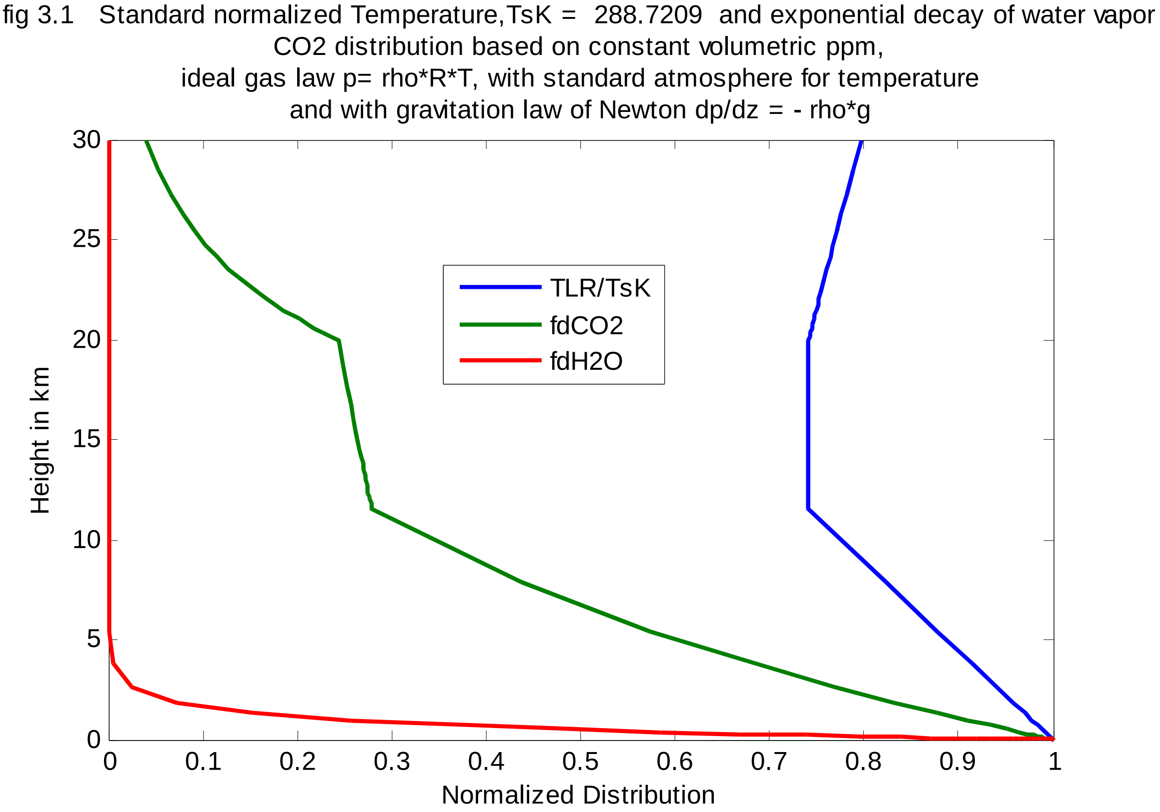


fig 3.1 Standard normalized Temperature,TsK = 288.7209 and exponential decay of water vapor
CO2 distribution based on constant volumetric ppm,
ideal gas law p= rho*R*T, with standard atmosphere for temperature
and with gravitation law of Newton dp/dz = - rho*g

The temperature distribution corresponds to the standard atmosphere with the three zones :

- the lower zone with the Environmental Lapse Rate $L_1 = - 6.5$ °C/km up to 11.5 km
- the isotherm middle zone with zero lapse rate $L_2 = 0$ between 11.5 km and 20 km
- the upper zone with a positive lapse rate $L_3 = 1.6$ °C/km

From this temperature distribution, using the ideal gas law and the gravitation law of Newton we can define the density variation of the air in the atmosphere.
Since the volumetric ppm of CO2 is constant one can define the mass concentration variation of CO2 in the atmosphere.

NB *Lapse rates are usually defined as* $\boldsymbol{\Gamma = -dT/dz}$, *resulting in a positive number for the lower atmosphere* $\boldsymbol{\Gamma = 6.5}$ ***ºC/km*** *(Wikipedia, lapse rates). The reason is that in earlier times very often elementary graphical procedures were used in meteorological analyses: negative slopes are not straight forward to use in such elementary graphical procedures.*

In computer programming it is more convenient to define a variation of temperature similar to the Taylor expansion **T = TsK +L*z**, where **TsK** surface temperature in °K, **z** the vertical coordinate above the surface z=0 and **the newly defined lapse rate L = dT/dz**.

Dry Adiabatic Lapse Rate, DALR

We consider the **Ideal Gas Law:**

$$\mathbf{p/\rho = RT} \quad \text{with} \quad \mathbf{R = c_p - c_v}, \quad \mathbf{v = 1/\rho} \quad \longrightarrow \quad \mathbf{pv = (c_p - c_v)\,T} \qquad \mathbf{(A2.1)}$$

p	pressure	$[N/m^2]$
ρ	density	$[kg/m^3]$
v	specific volume	$[m^3/kg]$
T	absolute temperature	[º K]
R	specific gas constant	[Joule/kg/ºC]
$\mathbf{c_p}$	specific heat for constant pressure	[Joule/kg/ºC]
$\mathbf{c_v}$	specific heat for constant volume	[Joule/kg/ºC]

In differential form the ideal gas law can be written as:

$$\mathbf{vdp + pdv} = \mathbf{(c_p - c_v)dT} \qquad \mathbf{(A2.2)}$$

Two equivalent expressions result for the First Law of Thermodynamics:

(a) $\mathbf{dQ = c_p\,dT - vdp}$ or **(b)** $\mathbf{dQ = c_v\,dT + pdv}$

dQ external heat supply [Joule/kg]

The two expressions are equivalent: when subtracted from each other the **dQ** term drops and the differential formulation of the ideal gas law results. When no external heat is present, the First Law **(a)** can be written in adiabatic version with **dQ** = 0:

$$c_p\,dT - vdp = 0 \qquad \text{(A2.2a)}$$

Gravitation law of Newton

$$dp/dz = -\rho g \quad [\,N/m^2\,] \quad \longrightarrow \quad dp = -\rho g dz \qquad \text{(A2.3)}$$

The combination of **(A2.2a)** and **(A2.3)** gives the dry adiabatic lapse rate:

$$DALR = dT/dz = -g/c_p \qquad \text{(A2.4)}$$

With $g = 9.81\ m/sec^2$ and $c_p = 1000\ J/kg/°C$: **DALR = - 9.81 °C/km**
The *measured* lapse rate outside the polar regions, the so-called Environmental Lapse Rate is:

$$ELR = -\,6.5\ \ [°C/km] \qquad \text{(A2.5)}$$

In the polar regions the lapse rate is closer to the dry adiabatic lapse rate **DALR**.

It can be concluded that the lapse rate for the bigger part of the planet outside the polar regions up to the equator remains equal to **ELR,** at least in the lower part of the atmosphere up to 11.5 km high.
In the analyses, the Standard Atmosphere as depicted in **figure A2.1** it has been applied.

CO2 density variation in the atmosphere

The volumetric concentration of CO2 in ppm on the planet Earth is more or less constant, from the equator towards the polar regions and from the surface of the planet up to a height of 30 km and higher.
From the **Ideal Gas Law** we find:

$$p = \rho * R * T \quad \longrightarrow \quad dp/dz = R*T*d\rho/dz + \rho*R*dT/dz$$

Inserting the gravitation law of Newton;

$dp/dz = -\rho*g$ one obtains: $R*T*d\rho/dz = -\rho*(R*dT/dz+g)$

In case in a section with starting values, z_1 and T_1 and lapse rate $L= dT/dz$, which can be positive (zone 3) or negative(zone 1):

$$d\rho/\rho = -(1+g/(R*L))*(L*d(z-z_1)/(T_1 + L*(z-z_1))$$

Integrating gives:

$$\ln(\rho) = -(1+g/(R*L))*\ln(T_1 + L*(z-z_1)) + \text{constant}$$

Or, for a region starting at T_1, z_1, ρ_1 and lapse rate L , the density ρ at a bigger z coordinate with temperature $T = T_1 + L*(z-z_1)$ can be written as:

$$\rho/\rho_1 = (T/T_1)\text{\textasciicircum}(-(1+g/(R*L)) \qquad \text{(A2.6)}$$

When $L = dT/dz = 0$, the temperature T is constant and we obtain;

$$d\rho/\rho = -g/(R*T)*dz.$$

Integrating gives: $\ln(\rho) = -(g/(R*T)*z + \text{constant.}$

Or for a region starting with ρ_1 and a height z_1:

$$\rho/\rho_1 = e\text{\textasciicircum}(-g/(R*T)*(z-z_1)) \qquad \text{(A2.7)}$$

The curve for normalized CO2 distribution in **figure A2.11** is obtained by applying for the three zones:

- Equation **(A2.6)** for the nodes in zone 1 with lapse rate L_1
  $\rho 1 = 1$, $T_1 = TsK$ and $z_1 = 0$.
  The last node of zone 1 (**nods1**) becomes the first node of zone2.
- Next, equation **(A2.7)** is applied for the nodes in zone 2 with
  $\rho_1 = \rho(nods1)$, calculated in zone1, and the constant temperature
  $T = T(nods1)$
  $z_1 = znod(nods1)$

- Next, equation **(1)** is applied for the nodes in zone 3 with

$\boldsymbol{\rho_1}$ = **ρ(nods1+nods2) , c**alculated in zone 2 , and

$\mathbf{z_1}$ = **znod(nods1+nods2)**

T = **T(nods1) +$\mathbf{L_3}$ * (z- $\mathbf{z_1}$)**

In the curve of **figure A2.1** for the normalized CO2 density distribution, the three zones are clearly recognized.

For the water vapor density distribution an empirical exponential relation is used:

H2O(i)=exp(-mH2O*znod(i)/5000) **(A2.7)**

The parameter **mH2O = 7** is defined by comparison of the stack results with K&T diagrams of mainstream authors.

Appendix 3

In this appendix are copied the m.files of the layersjan25.m main MATLAB program.

In MATLAB listings, a sentence with a ‘%’ sign in front is considered as a comment. It is printed in green.

The user who has difficulties to download these files on a PC with MATLAB program installed, may contact the author for new files jwreynen@gmail.com

```
  %layers20jan2025.m

%     Various analyses can be carried out by the MATLAB 
program:
%
%  option(1)
%     Calculation of watervapor heat fluxes q = K*theta 
with theta = sigma*(TsK+L*z)^4,
%     where TsK+L*z is the air temperature distribution, 
with lapse rate L=dT/dz.
%     NB Environmental Lapse Rate is defined as ELR = 
-dT/dz. For more transparent programming
%        the definition L = dT/dz is more logical
%     The analyses are based on the Stefan-Boltzmann-
Christiansen relation from 1883
%     for the one-stream heat transfer between two plates 
with different surface conditions
%     The calculated fluxes q are:
%         q(1): the first component is the LW surface 
flux, including the flux
%               through the atmospheric window, directly 
towards outer space
%     -q(nods): the last component represents the 
outgoing LW radiation: OLR
%
%     The remaining components are the necessary heat 
deposits by mechanisms
%     other than LW radiation to compensate for the loss 
of heat due to LW
%     emission by the traces of IR-active gases!
%     The air temperature distribution is given by the 
lapse rate L =dT/dz and
%     the surface temperature.
```

```
%     Figures are given of LW surface flux and OLR
(outgoing LW radiation)as function of ftot.
%     The difference between OLR and qsurf is the heat
deposited in the
%     atmosphere by convection of active and latent heat
and the heat
%     due to absorption of SW incoming radiation.
%
%     option 2
%     A demonstration of the saturation phenomenon with %
hypothetical water
%     vapor with ftot >1.
%
%     option (3)
%      The saturation phenomenon of CO2 in the atmosphere
is demonstrated.
%       CO2 thermalization of 18 Watt/m2 accordingto
Pangburn is taken into account
%      CONCLUSION: CO2 is not warming the atmosphere
%
%   Make a choise in the following MATLAB program

option = 4            %With this value for option the
loop is entered with three possible options

while option > 0       % outer loop N
clear                  %loop over the total program,
until option =0 or > 3
disp('                LIST OF OPTIONS')
disp('option == 1:    Watervapor analysis')
disp('option == 2:    Demonstration of non-existing
saturation of Watervapor')
disp('option == 3:    CO2 sensitivity analysis with
saturation and thermalization ')
disp('   ')
iter=1;              %in the water model iterations are
carried out to find a solution
                     %for OLRH2O =240-OLRCO2.
while iter>0
option=input('Make a choice for an option, finish with
0:')
  if option < 4
    if option>-1
```

```
        iter=0;
    end
  end
     if iter==2
     iter=0;
     option=0;
     end
     if iter == 1
     disp('Option should be between 0 and 4')
     disp('Hit any key and try once more')
     pause
     iter=iter+1
     end
end
disp('input for options finished')

R  = 287.058;         %= cp-cv ideal gas  constant per
J/kg
cp = 1000;            % specific heat of air at constant
pressure J/kg
g  = 9.81;            % gravitation acceleration m/s^2
TsC = 15.7209;        % surface temperature in C
TsK = TsC+273;        % surface temperature in K
sigma=5.67*10^-8;    % Stefan-Boltzmann constant
W/m^2/K^4
                       % the influence of the lapse rate,
                       % see appendix  in [1] for a
derivation from
                       %the ideal gas law and Newtons law
for gravitation.
L  = -6.5/1000;        % LdT/dz environmental lapse rate
-6.5 K/km
                       %in meteorpology lapse rates are
taken positive
                       % because of the graphical
procedures in that field.
                       %in the computer software it is
better to use the
                       %derivative  as the slope Lapse
rate are equal to
                       % L= dT/dz = -6.5 C/km and
converted to C/m.
```

```
mH2O = 7;               % coefficient for the exponential
distribution of water vapor
                        % users are invited to play with
it
                        % mH2O =7 turned out to give
results corresponding to
                        % experimental results of
Miskolczi publications
                        % see ref 7

qtoa  = 230.176;         %water vapor  flux at top of
atmosphere according to NASA, W/m^2
                        %240 - 9.824 (= CO2qtoa) from
runs with the present software
                        % used in plots for finding
reference point, see K&T
                        % diagrams in reference [1,2,3

qPrevost0 = sigma*TsK^4; %the heat flux to outer space
without infra-red-actif gas
                         %like water vapor or CO2 gas.
                         % theta =sigma*T^4 as heat flux
or as temperature parameter
fractionCO2 =46/qPrevost0;  %Pangburn figure from NASA
for TsK =294 corrected to 46 see [1]
fractionH2O=1-fractionCO2;
                          %the CO2 notch represent a
fraction fractionCO2 of the total spectrum
                          %this value is taken to show
the saturation phenomenon
                          %water vapor, water droplets
water and other particles

znodg =zeros(4,1);        %data temperage distibution up
to 30 km for CO2 analysis
Tnodg=zeros(4,1);         %standard atmosphere
znodg(1)=0;
znodg(2)=11500;
znodg(3)=20000;
znodg(4)=30000
Tnodg(1)= TsK;
Tnodg(2)= Tnodg(1)+L*(znodg(2)-znodg(1));
Tnodg(3)= Tnodg(2);
```

```
Tnodg(4)= 230;


if option < 3              % the water analyses in option 1
and 2, CO2 analysis
                           % for option = 3.

if option ==1               %basic water vapor H2O analysis

givenftot = 0.81;            %this value is calculated by
earler runs of the stackmodel
                            %and based on the general
accepted average
                             %outgoing LW flux:OLR=240 W/m2
nabsorb = 10;                %the value nabsorb control de
output figures.
                             % it are the number of
analyses  as function of ftotH2o
end
if option == 2              %to show the saturation
phenomenon in watervapor
    givenftot=1.5;          %for ftotH2O >1 also water vapor
in the air saturates for heat absorption
                            % 10 analyse between ftotH2O =0
to 1.
    nabsorb1 = 10
    nabsorb  = 15;            % 20 -10 analyses for ftotCo2
=1 to 1.5
end

fp    = zeros(nabsorb,1); %emissio/absorption
coefficients values for which analyses are carried out
                             %graphical results

if option ==1                %nabsorb values to be
defined(values for which analyses are
                            %carried out for water vapor
analyses
ftot =givenftot;            %for option 1 and 2 it is also
called ftotH2O
fmin=0.001;
fmax = ftot;
 for jf=1:nabsorb
```

```
     fp(jf)= fmin+(jf-1)*(fmax-fmin)/(nabsorb-1);
 end
end

if option == 2          %special watervapor analyses to
show that for fotH2O >1
                        %saturation occurs also for
watervapor.
ftot =givenftot;
fmin=0.001;
fmax = 1;
for jf=1:nabsorb1
     fp(jf)= fmin+(jf-1)*(fmax-fmin)/(nabsorb1-1);
end
fp(nabsorb1)=1;
 fmin=1;
 fmax=givenftot;
  for jf=nabsorb1:nabsorb
        fp(jf)= fmin +(jf-nabsorb1)*(fmax-fmin)/(nabsorb-
nabsorb1);
  end
  fp(nabsorb)=givenftot;

end        %end of opton 2
                              %nabsorb values to be
defined(values for which
                              %analyses are carried out
for water vapor

qsurf   = zeros(nabsorb,1);    %LW surface flux, from
surface into atmosphere,
qwindow = zeros(nabsorb,1);     %it includes qabsorb and
qwindow
OLR     = zeros(nabsorb,1);     %Outgoing Longwave
Radiation
qz      = zeros(nabsorb,1);     %heat input other than LW
surface radiation
qtoav   = zeros(nabsorb,1);    %make a vector of qtoa and
Prevost
qPrevostv =zeros(nabsorb,1);   %for plotting
purposesqabsorb  = zeros(nabsorb,1);
nods = 50;
qzv      = zeros(nabsorb,nods);
```

```
thetav   = zeros(nabsorb,nods);
tempv    =  zeros(nabsorb,nods);

              % nodes in the atmosphere for water vapor
analysis,
                         % including surface node=1 and
outer space node = nods

heightkmH2O=11.5;      % default height of H2O model in
km, can be changed
                       %for water vapor 7km would be
sufficient, for CO2 we
                       %use 25

toakm    = heightkmH2O;
heightH2O= heightkmH2O*1000;  % convert data from km into
meters

%mesh generation parameters
ratioatm =1.23 ;          % ratio for the geometrical
series of distribution of nods.
                         % for the distribution of IR-
active trace gases.

 znod=zeros(nods,1);   %nodal coordinates for watervapor
analysis,
                      %ground level at znod=0
 znod(1) = 0;
 for i=2:nods
    znod(i)=znod(i-1)+ratioatm^(i-1);
 end
 ratio=heightH2O/znod(nods);
 znod =ratio*znod;  %coordinates are now between 0 and
heightH2O

dzv =diff(znod);        %diff Matlab function to make a
vector of differences
dzv;                       %in fact the element sizes
dzv1=dzv(1);
disp ('dzv1: element size in meter at bottom znod=0')
dzv1
dzvnods =dzv(nods-2)/1000;
```

```
disp('dzvnods: element size in km at top of atmosphere
11.5 km ')
dzvnods
TLR=zeros(nods,1);                %temperature
distribution in Kelvin
theta=zeros(nods,1);              %alternative variable
for temperature

%theta(i)=sigma*TLR(i)^4
dthetadTsK=zeros(nods,1);         %derivataltернative
variable for temperature
for i=1:nods-1
    znodi=znod(i);
    TLRi=TsK+L*znodi;
    TLR(i)=TLRi;                     % temperature
distribution  for TsK and L
    theta(i)=sigma*TLRi^4;           % theta are
convenient parameters for temperature
    dthetadTsK(i)=4*theta(i)/TLRi;   %to be used for
dOLR/dTsK
end
dthetadTsK;
qPrevost=fractionH2O*theta(1);   % Prevost surface flux
TLR0=TLR;
zeroK= 0 %2;               % outer space taken as zero,
                           %can also be taken as 2 degree
K,
                           %no difference in answers!
theta(nods)=zeroK;              % sigma*zeroK^4;
znod(nods)=znod(nods-1);


f=zeros(nods,1);
f(1)=1;   %fractionH2O;    %emission/absorption
coeeficients  of the surface
f(nods)=1;       %outer space f

% distribution of absorption at nods = znod(i)
% exponential decay of absorption with heightH2O defined
by mH2O
% for mH2O=0 homogeneous distribution, mH2O=7 gave the
best comparison with the
% experimental results of K&T diagrams
```

```
% distribution of CO2 proportional to density

for i=2:nods-1
    znodi=znod(i);
    zref=5000;
    H2Oratio=exp(-mH2O*znodi/zref);
    fd(i)=dzv(i-1)*H2Oratio;
end

fd(1)   =0;                      % at node =1 (surface)
and at node =nods (outerspace)
fd(nods)=0;
totfd   =sum(fd);                %normalized distribution
of absorption, total normalized = 1
fd=fd/totfd;

f(1)=1;                         %fractionH2O taking take
care of later
dzv =diff(znod);        %diff Matlab function to make a
vector of differences
dzv1=dzv(1);
dzvnods=dzv(nods-1);

fd(1)  = 0;                      % at node =1 (surface)
and at node =nods (outerspace)
fd(nods)= 0;                   % make fd=0 for both H2O
and CO2
                              %normalized distribution of
absorption, total
                              %normalized = 1
totfd=sum(fd);
fd=fd/totfd;
for i=1:nods
    theta(i)=sigma*TLR(i)^4;   %sigma*T^4
end
totfd=sum(fd);
fd=fd/totfd;


jf=0;
mismatch=0;                   %mismatch is difference
with target in iteration
dfj=0;
```

```
qPrevost=fractionH2O*sigma*TsK^4;   % Prevost surface
flux without infra-red-active gases

jf=0;
while jf<nabsorb               % loop through water
absorption levels for options 1, 2, 3
  jf=jf+1;
  qtoav(jf)=qtoa;               %for plotting a
horizontal line=qtoa
  qPrevostv(jf)=qPrevost;       %for plotting a
horizontal line = qPrevost
  f=zeros(nods,1);
  fpjf=fp(jf);
  f=fd*fpjf;
  f(1)= 1;                        %surface coefficient
f(1)==1
  f(nods)=1;                      % outer space f(nods)=1
  qwindow(jf)= (1-fp(jf))*qPrevost;
  if qwindow(jf) < 0
      qwindow(jf)=0;
  end
     makeviewfactor
     makeK              %K matrix
     K=fractionH2O*K;   %in fact fractionH2O is applied
to theta

    theta(nods)=zeroK;  %outer space temperatrure at
zeroK, just to be sure  in a
                        %previous plotting option the
value is set to
                        %theta(nods-1) for plotting
reasons
    q=K*theta;          % q is includes contribution from
mechanisms other than radiation
                        %except first term = qsurf and
last term=-OLR
    qwindowjf=(1-fp(jf))*fractionH2O*theta(1);
    if qwindowjf <0
        qwindowjf=0;
    end
    qwindow(jf)=qwindowjf;
    qsurf(jf)=q(1);     %outgoing radiation due to LW
surface flux,
```

```
                        %including flux through window:
qwindow(jf)
    OLR(jf)=-q(nods);   % total outgoing including
mechanisms
                        %other than surface radiation

   if option==1
                        %correct for plotting in a non-
homogeneous mesh, qinto and qout are
                        %nodal values, W/m^2, to be
converted to volumetric values W/m^3.
                        %The difference is due to
mechanisms other than LW radiation
     makeqz             % deposit mechanisms other than
LW radiation from surface

     qzv(jf,:)= qz;    %z-distribution of LW for
different absorbtions ftot
   end

   OLR(jf) = -q(nods);
   OLRpr=qPrevost;
   if jf >nabsorb-2
     OLRpr=OLR(jf-1);
     dOLRdftot =(OLR(jf)-OLRpr)/(fp(jf)-fp(jf-1));
   end
   qsurf(jf) = q(1);
   qwindowjf=qwindow(jf);

 if jf<nabsorb
     iter=0;
 end
 if option== 1
   if jf==nabsorb
    disp('Iterations to obtain the ftot value for OLR
=qtoa =  240  -CO2 contribution ')
    iter =iter+1
    OLRjf=OLR(jf);
    mismatch=OLRjf-qtoa;
    dfj= - mismatch/dOLRdftot;
    fpnab =fp(nabsorb);
    fpnab = fpnab+dfj;
    fp(nabsorb)= fpnab;
```

```
    if abs(mismatch/qtoa)>0.0001
    jf=jf-1;
    else
        jf=nabsorb;
    end
    if iter>9
       disp('More than 10 iterations for qtoa')
       jf=nabsorb;
    end
   end
 end
end
disp('end of nabsorb loop')

dqdTsK      =   zeros(nods,1);
dqdTsK      =   K*dthetadTsK;
dOLRdTsK    = - dqdTsK(nods)
dqsurfdTsK = dqdTsK(1);
OLRH2O=-q(nods);
qsurfH2O=q(1);
 maxftot=fp(nabsorb);
 maxftot= round(maxftot*10000)/10000;
 fp(nabsorb)=maxftot;
 ftot=maxftot;
  disp('Input for global K&T type heat budget')
  OLRftot=OLR(nabsorb)
  qsurfftot=qsurf(nabsorb)
  qwindowftot=qwindow(nabsorb)

 end        %end of loop for option<3 for water vapor
analyses

 if option == 3          %CO2 analyses as compared to the
water vapor analysis
                       %often the same variabkle names
are used, however
                       %when plotting the resulte "CO2"
is added to the
                       %variable names.
 disp('Detailed CO2 analysis for saturation and
thermaliztion');
 disp(' for values between ftotCO2=0 and ftotCO2=
givenftotCO2')
```

```
 disp('                 LIST OF givenftotCO2 values')

disp('givenftotCO2 = 1: saturation value')
disp('givenftotCO2 = 2: twice   the saturation value ')
disp('givenftotCO2 = 3: three times saturation value')
disp('givenftotCO2 = 4: four times  saturation value')
disp('   ')
iter=1;
while iter>0
givenftot=input('Make a choice for a givenftot finish with 0:')
  if givenftot < 4.1
    if givenftot>0.99
        iter=0;
    end
  end
     if iter==2
     iter=0;
     givenftot=0;
     end
     if iter == 1
     disp('Givenftot should be between 1 and 4')
     disp('Hit any key and try once more')
     pause
     iter=iter+1
     end
end

     %INPUT FOR OPTION 3

   switch givenftot
    case {1}                %plotting for ftotCO2=1
        nabsorb =20
    case {2}                %plotting for ftotCO2=2
        nabsorb =40
    case {3}                %plotting for ftotCO2=3
        nabsorb =50
    case {4}                %plotting for ftotCO2=4
        nabsorb =60
   end

   nods=100
   fp       = zeros(nabsorb,1);
```

```
   qwindow   = zeros(nabsorb,1);    %it includes qabsorb
and qwindow
   OLR       = zeros(nabsorb,1);    %Outgoing Longwave
Radiation
   qz        = zeros(nabsorb,1);    %make continuous curve
of nodal value q(i)
   qtoav     = zeros(nabsorb,1);    %make a vector of qtoa
and Prevost
   qPrevostv=zeros(nabsorb,1);      %for plotting purposes
   qzv       = zeros(nabsorb,nods);

  OLRtherm    = zeros(nabsorb,1);
  deltaTsK    = zeros(nabsorb,1);
  forcingOLR = zeros(nabsorb,1);
  dOLRH2OdTsKv =zeros(nabsorb,1);

if givenftot == 1
fmin=0.001;
fmax = 1;
for jf=1:nabsorb
     fp(jf)= fmin+(jf-1)*(fmax-fmin)/(nabsorb-1);
end
fp(nabsorb)=1;
end
if givenftot>1
   nabsorb1=10
   fmin=0.001;
   fmax = 1;
   dftot= (fmax-fmin)/(nabsorb1-1);
   for jf=1:nabsorb1
     fp(jf)= fmin+(jf-1)*(fmax-fmin)/(nabsorb1-1);
   end
   fp(nabsorb1)=1
   fmin=1
   fmax=givenftot;
   for jf=nabsorb1+1:nabsorb
        fp(jf)= fmin +(jf-nabsorb1)*(fmax-fmin)/(nabsorb-
nabsorb1);
   end
   fp(nabsorb)=givenftot;
end

    CO2    = zeros(nods,1);
```

```
    H2O   = zeros(nods,1);
    fd    = zeros(nods,1);
    znod  = zeros(nods,1);
    q     = zeros(nods,1);
    viewfactor = ones(nods);
    TLR   =zeros(nods,1);
    theta =zeros(nods,1);            %alternative variable
for temperature
    qPrevost0 = sigma*TsK^4;         % Prevost flux no
water vapor and no CO2 gas
    qPrevost=fractionCO2*qPrevost0; %Prevost in presence
of CO2 gas,
                                     %is plotted as
PrevostCO2
    qtherm = 18;                     %fraction of Prevost
flux thermalized\
                                     %according to
Pangburn 18 out of 46 W/m2


 meshCO2           %  temperature and theta CO2
distribution are calculated
                   %including plotting of distributions of
H2O and CO2
                   %densities

dzv=diff(znod);    %from ndal values for z-coordinate make
element sizes dz
for i=2:nods-1
  fd(i)=dzv(i-1)*CO2(i);
end
fd(1)=0;                            %normalized distribution of
absorption, total normalized = 1
                            % at node =1 (surface) and at
node =nods (outerspace)
fd(nods)=0;

totfd=sum(fd);
fd=fd/totfd;
dzv1=dzv(1);
dzvnods=dzv(nods-1);
```

```
dOLRH2OdTsK= 3.2342;          %from previous runs option
1
heightkmCO2 = round(znodg(4)/1000);

   fmin=0.0001;
   fmax=1;
   if givenftot<1.01
       nabsorb1=nabsorb;
   end
   for jf=1:nabsorb1
        fp(jf)= fmin +(jf-1)*(fmax-fmin)/(nabsorb1-1);
   end

   if givenftot>1
   fmin =1;
   fmax =givenftot;
   for jf =nabsorb1:nabsorb
       fp(jf)=fmin+(jf-nabsorb1)*(fmax-fmin)/(nabsorb-
nabsorb1);
   end
   fp(nabsorb)= givenftot;
   end
   jf=0;

   while jf<nabsorb
   jf=jf+1;
   K = zeros(nods);
   f=zeros(nods,1);
   fpjf = fp(jf);
   for i=2:nods-1
      f(i)=fd(i)*fpjf;
  end
  f(1)= 1;          %surface f(1)=1

  f(nods)=1;         % outer space f(nods)=1

   % assemblage of system matrix K
   makeviewfactor
   makeK
   K = fractionCO2*K;
   q = K*theta;
   qsurfjf= q(1);
   OLRjf = -q(nods);
```

```
   qPrevostv(jf)=qPrevost;
   forcingOLR(jf) = qPrevost-OLRjf;
   deltaTsK(jf) =forcingOLR(jf)/dOLRH2OdTsK;
   OLR(jf)  = OLRjf;
   qsurf(jf)= qsurfjf;
   if qtherm > 0                         %to include the
thermalisation of CO2 according to Pangburn 18 W/m2
       qthermcor=qtherm;
       if fpjf<1.002
       qthermcor=fp(jf)*qtherm;          % and constant
between ftotCO2 = 2 to 4.
                                         %thermalisation
distribution linear between ftorCO2 =1 to 2
       end
       OLRthermjf=OLRjf+qthermcor;       %it could be that
the OLRtherm near ftot = 0 is bigger than qPrevost
       if OLRthermjf>qPrevost
           OLRthermjf=qPrevost;
       end
       qsurfthermjf=qsurfjf+qthermcor;
       if qsurfthermjf>qPrevost
           qsurfthemjf=qPrevost;
       end
       OLRtherm(jf)= OLRthermjf;
       qsurftherm(jf)=qsurfthermjf;
       forcingOLRtherm(jf) = qPrevost-OLRthermjf;
       deltaTsKtherm(jf)
=forcingOLRtherm(jf)/dOLRH2OdTsK;
   end

   dOLRH2OdTsKv(jf)=dOLRH2OdTsK;
   qwindowjf=(1-fpjf)*qPrevost;
   if qwindowjf<0
       qwindowjf=0;
   end
   qwindow(jf)=qwindowjf;
 end
 disp('end of nabsorb loop')

 ftotCO2=fp(nabsorb);
 dOLRH2OdTsK                %defined in wateranalysis
 maxftot = fp(nabsorb);
```

```
q=fractionCO2*K*theta;
OLRCO2=-q(nods);
qdisCO2=sum(q)-q(1)-q(nods);
qsurfCO2=q(1);
makeqz
disp('Input for global heat budget')
  OLRftot=OLR(nabsorb);
  qwindowtot = qwindow(nabsorb);
  qsurfftot=qsurf(nabsorb);
 end
switch option
    case {1}
        plotoption1
    case {2}
        plotoption2
    case {3}
        plotoption3
 end
end

%plotoption1.m

xtitle1a = [' nods = ',...
        num2str(nods), '  mH2O = ', num2str(mH2O),'
heightH2Okm = ',...
        num2str(heightkmH2O),' LdT/dz = ',
num2str(1000*L) ' degree/km',' TsK =  ', num2str(TsK)];

    xtitle1b = [ ' ftotH2O =  ',num2str(maxftot),'
fractionH2O =  ',num2str(fractionH2O),...
                 ' qtoaH2O = ',num2str(qtoa),'
qPrevostH2O = ',num2str(qPrevost)];
    xtitle1c = [  'OLRH2O, qsurfH2O as function of
ftotH2O. qwindowH2O = (1-ftotH2O)*qPrevostH2O'];

  xtitleH2O = [' Result of water vapor analysis '];

  figure                    %fig1.1

plot(fp,OLR,fp,qsurf,fp,qwindow,fp,qPrevostv,':',fp,qtoav
,':','LineWidth',2)
  xtitle = ['fig 1.1     Result of H2O analysis with nods
=',num2str(nods),];
```

```
  title({xtitle;xtitle1a;xtitle1b;xtitle1c;})

legend('OLRH2O','qsurfH2O','qwindowH2O','qPrevostH2O','qt
oaH2O')
  xlabel('ftotH2O')
  ylabel('W/m^2')


  figure     %fig 1.2

  plot(qz,znod/1000,'LineWidth',2)   %distribution of
heat deposits due to the imposed L
  xtitle = ['fig 1.2 Result of H2O analysis with nods
=',num2str(nods), '  q = K*theta'];
  xtitle1a = [  ' Distribution q(2) up to
q(',num2str(nods-1),'): heat imput by other than'...
                'LW radiation,  such as;'];
  xtitle1b = [  ' convection, SW absorption, CO2
thermalization etc.Total qtot W/m^2 '];
  title({xtitle;xtitle1a;xtitle1b;})
  str1=  [ ' qtot= ',num2str(qtot),'  W/m^2' ];
  legend(str1)
  xlabel('Necessary heat input per m^3 other than LW
radiation  W/m^3')
  ylabel('HeightH2O in km')


%plotoption2.m
figure % fig 2.1
     plot(fp,OLR,fp,qsurf,fp,qwindow,...
                         fp,qPrevostv,':','LineWidth',2)
     xtitle = ['fig 2.1    Result of H2O analysis with
nods =',num2str(nods),];
     xtitle1= [' Watervapor radiation saturation for
ftotH2O > 1 '];
     title({xtitle;xtitle1})

legend('OLRH2O','qsurfH2O','qwindowH2O','qPrevostH2O')
                                   %when plotting results for
watervapor "H2O" is added to the variable names
     xlabel('  ftotH2O ','LineWidth',2)
     ylabel('W/m^2','LineWidth',2)
```

```
%plotoption3.m


    figure      %fig 3.1 input data for watervapor, CO2
density and temperture
                %as function of height.


plot(TLR/TsK,znod/1000,CO2,znod/1000,H2O,znod/1000,'LineW
idth',2)
    xtitle1 = ['  Standard normalized Temperature,TsK =
',num2str(TsK),...
           '  and exponential decay of water vapor'];
    xtitle2 = ['CO2 distribution based on constant
volumetric ppm,'];
    xtitle3 = ['ideal gas law p= rho*R*T, with standard
atmosphere for temperature']
    xtitle4 =['and with gravitation law of Newton dp/dz =
- rho*g']
    xtitle = ['fig 3.1 ',xtitle1];
    title({xtitle;xtitle2;xtitle3;xtitle4;})
    legend('TLR/TsK', 'fdCO2','fdH2O')
    xlabel('Normalized Distribution')
    ylabel('Height in km')



  xtitle1 = ['fig 3.2  Result of CO2 analysis with nods
=',num2str(nods), '   Without thermalisation '];
  xtitle2 = ...
        [  '   heightCO2km =',...
        num2str(heightkmCO2),' ftotCO2
=',num2str(maxftot),...
     ' TsK =', num2str(TsK),' K  fractionCO2
=',num2str(fractionCO2),' qPrevost =',num2str(qPrevost)];
  xtitle3 = ['dOLRH2OdTsK =  ',num2str(dOLRH2OdTsK),'
deltaTsK = forcingOLR/(dOLRH2OdTsK)',...
             '    qtherm= 0 ']
  xtitle4 = ['qtherm=  ',num2str(qtherm)];
  figure                                         %without
thermalisation qtherm==0
```

```
  plot(fp,OLR,fp,qsurf,fp,qwindow,fp,qPrevostv,':',...

fp,forcingOLR,fp,deltaTsK,'LineWidth',2)
  title({xtitle1;xtitle2;xtitle3});

legend('OLRCO2','qsurfCO2','qwindowCO2','qPrevostCO2','fo
rcingOLR','deltaTsK')
  xlabel('ftotCO2','LineWidth',2)
  ylabel('W/m^2     degreeK','LineWidth',2);

  if qtherm>0
  figure                                         % with
thermalisation  qtherm >0

plot(fp,OLRtherm,fp,qsurftherm,fp,qwindow,fp,qPrevostv,':
',...

fp,forcingOLRtherm,fp,deltaTsKtherm,'LineWidth',2)
  xtitle1th = ['fig 3.2therm   Result of CO2 analysis
with nods =',...
                num2str(nods), '   With thermalisation
'];
  xtitle3 = ['dOLRH2OdTsK =  ',num2str(dOLRH2OdTsK),...
             '  deltaTsK = forcingOLR/(dOLRH2OdTsK)','
qtherm=  ',num2str(qtherm)];
  title({xtitle1th;xtitle2;xtitle3});

legend('OLRCO2+qtherm','qsurfCO2+qtherm','qwindowCO2',...
         'qPrevostCO2','forcingOLR','deltaTsK')
  xlabel('ftotCO2','LineWidth',2);
  ylabel('W/m^2     degreeK','LineWidth',2);
  end

  figure      %fig 3.3

plot(fp,OLR,fp,qsurf,fp,qwindow,fp,qPrevostv,':','LineWid
th',2)
  xtitle1 = ['fig 3.3    Result of CO2 analysis with nods
=',...
              num2str(nods),'   Without
thermalisation' ];
  title({xtitle1;xtitle2});
  legend('OLRCO2','qsurfCO2','qwindowCO2','qPrevostCO2');
```

```
  xlabel('ftotCO2','LineWidth',2);
  ylabel('W/m^2','LineWidth',2);

  figure    %fig3.4 Heat exchange other than LW radiation
with atmosphere
            %as function of height in W/m^3

  plot(qz,znod/1000,'LineWidth',2)

  xtitle1 = ['fig 3.4  Result of CO2 analysis' ];
  xtitle3a = [  ' Distribution of necessay heat deposit
by '...
                'other than LW radiation,  such as;'];
  xtitle3b = [  ' convection, SW absorption etc.Total
qtot W/m^2 '];
  title({xtitle1;xtitle2;xtitle3a;xtitle3b});
  str1=  [ ' qtot= ',num2str(qtot),'  W/m^2' ];
  legend(str1);
  xlabel('Necessary heat input per m^3 other than LW
radiation  W/m^3');
  ylabel('Height in km')



%makeqz
qz=zeros(nods,1);
for i=2:nods-1
    qz(i)= q(i)*2/(znod(i+1)-znod(i-1));
end
qz(1)=qz(2);
qz(nods)= qz(nods-1);
qtot=sum(q)-q(1)-q(nods);

 %makeK
K = zeros(nods);
fe=zeros(nods);
for i=1:nods-1
  fi1=f(i);
 for j=i+1:nods
  fi=fi1*viewfactor(i,j);
  fj=f(j);
  %Christiaansen
  feij=1/(1/fi+1/fj-1);
```

```
  fe(i,j)= feij;
  makeKij;
 end
end

%meshCO2
%lower atmosphere;
znod(1)=0;
qtoaz(1)=qtoa;
dz=znods2/(nods2-1);
for i=2:nods2
    dz = 1.05*dz;
    znod(i)=znod(i-1)+ dz;
    qtoaz(i)=qtoa;
end
znodnods = znod(nods2);
znod =znod*znods2/znodnods;
TLR=zeros(nods,1);  %temperature distribution according
to laps rates LR in lower atmosphere
theta=zeros(nods,1);
 Tnods1=TsK
 Tnods2=Tnods1+ELR*znods2;
 Tnods3=Tnods2+0.1;     %otherwise ELR2 will be zero
 Tnods4=230;

    ELR1=ELR;
    ELR2=(Tnods3-Tnods2)/(znods3-znods2);
    ELR3=(Tnods4-Tnods3)/(znods4-znods3);
    zeroK=0;                            % outer space taken as
zero

    h2o(1)=1;
    co2(1)=1;

    TLR(1)=TsK;
for i=2:nods2
    znodi=znod(i);
    TLRi=TsK+ELR1*znodi;
    TLR(i) = TLRi;
    ratioH2O=exp(-m*znodi/5000);
    ratioCO2=(TLRi/TsK)^(-(g/R/ELR1+1));

    % ratioCO2=(TsK/(TsK+LR*znodi))^(g/R/LR+1);
```

```
                              %?/?s= 1/(1+ELR*z/Ts)^(1 +
g/ELR/R)=CO2ppm/400e-6


    h2o(i)=ratioH2O;
    co2(i)=ratioCO2;
end
ratioCO2old=ratioCO2;
%temperatures between nods2 and nod3
dz=(znods3-znods2)/(nods3-nods2);
for i=nods2+1:nods3
    znodi=znod(i-1)+dz;
    znod(i)=znodi;
    TLRi=Tnods2+ELR2*(znodi-znods2);
    TLR(i)=TLRi;
    ratioH2O=0;
    h2o(i)=ratioH2O;
    ratioCO2=ratioCO2old*(TLRi/Tnods2)^(-(g/R/ELR2+1));
    co2(i)=ratioCO2;
 end
ratioCO2old=ratioCO2;

%temperatures between nods3 and nods4
dz=(znods4-znods3)/(nods4-nods3);
for i=nods3+1:nods4
    znodi=znod(i-1)+dz;
    znod(i)=znodi;
    TLRi=Tnods3+ELR3*(znodi-znods3);
    TLR(i)=TLRi;
    ratioH2O=0;
    h2o(i)=ratioH2O;
    ratioCO2=ratioCO2old*(TLRi/Tnods3)^(-(g/R/ELR3+1));
    co2(i)=ratioCO2;
end
m
nods
height=znods4

for i=1:nods
    thetai=sigma*TLR(i)^4;
    theta(i)=thetai;
    dthetadT(i)=4*thetai/TLR(i);
end
```

```
theta(nods)=0;
dzv=diff(znod);
 for i=2:nods-1
     fdCO2(i)= dzv(i)*co2(i);
 end
fdCO2(1)=0;
fdCO2(nods)=0;
totfdCO2=sum(fdCO2);
fdCO2=fdCO2/totfdCO2;
```

```
%plotdistribution.m
xtitle1 = ['  Standard normalized Temperature,TsK =
',num2str(TsK),...
           '  and exponential decay of water vapor'];
xtitle2 = ['CO2 distribution based on constant volumetric
ppm,'];
xtitle3 = ['ideal gas law p= rho*R*T, with standard
atmosphere for temperature']
xtitle4 =['and with gravitation law of Newton dp/dz = -
rho*g']

    figure
    xtitle = ['fig 4.3 ',xtitle1];

plot(TLR/TsK,znod/1000,CO2,znod/1000,H2O,znod/1000,'LineW
idth',2)
    legend('TLR/TsK', 'fdCO2','fdH2O')
    title({xtitle;xtitle2;xtitle3;xtitle4;})
    xlabel('Normalized Distribution')
    ylabel('Height in km')
```

```
%makeviewfactor.m
viewfactor = ones(nods);

  for i=1:nods-2
    for j=i+2:nods
        viewfactorij=viewfactor(i,j);
```

```
           for k = (i+1):(j-1)
               if k>i
                   if k<j
            viewfactorij=viewfactorij-f(k);% FEM
viewfactor(i,j)=1-f(i+1)-f(i+2))... -f(j-1)
                   end
               end
           end
          if viewfactorij<0
            viewfactorij=0;
          end

        viewfactor(i,j)= viewfactorij;
        viewfactor(j,i)= viewfactorij;  %not necessary
because i always < j

    end
  end
```