Stack Model to Show the Benefit of Carbon Dioxide in the Atmosphere

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Summary

Today in the United States of America the 47th President has been installed.
One of the promises of the new President is to withdraw the US from the
2016 Paris agreement on Climate Change, for the second time.
It is to be expected that this time many other counties will follow the US.
The author has been skeptic on Climate Change for more than a decade, in
particular concerning the claim of the IPCC (International Panel on
Climate Change) that CO_2 gas causes the planet Earth to heat up.
There is not such a climate crises, it is a hoax. CO2 is plant food.

As a retired scientist, in the field of safety of nuclear reactors, the author
has used finite element techniques to analyze the LW (long wave) radiation
from the surface of the planet to send back to outer space the heat which
the the Sun is sending by SW (short wave) radiation to planet Earth.
The infra-red-active trace gases are simulated by a stack of fine gauze.
This paper with the **Appendices** repeats the basis of the stack model:
- phenomenons of thermalization and saturation of carbon dioxide gas CO_2
- the one-stream formulation of Stefan-Boltzmann-Christiansen of
  1883 for the LW radiation through water vapor gas and carbon dioxide
  gas which are the infra-red-active gases considered
-using the ideal gas law (PV=RT) to derive the natural adiabatic
  atmospheric temperature distribution with lapse rates of the so-called
  Standard Atmosphere, and the distribution of CO_2 gas in such standard
  atmosphere up to a height of 30 km
-listing of the .m files of the stack model which can be used on any PC
 with an installed MATLAB program.

This material can be used by any person with a science middle school
education, or better with the mathematics in first year of science faculties.

History of the stack model to analyze the evacuation of heat from the planet.

Infra-red-active gases in the atmosphere are mostly gases with 3 or more atoms per molecule: water-vapor (H_2O), carbon dioxide (CO_2), methane (CH_4), ozone (O_3), laughing gas (N_2O)... .
They are also known as greenhouse gases, a misnomer!
Infra-red-active gases hinder long-wave (LW) radiation to outer space from the surface of the planet, to evacuate the heat which the sun is sending to the planet as short-wave (SW) radiation with smaller absorption losses in aerosols.
The most important infra-red-active gases are H_2O vapor and CO_2 gas: they are the subject of this paper.
In earlier papers [1] the author has studied the hindering of the LW evacuation of heat from the terrestrial surface to outer-space by a mechanism of a stack of fine gauze, simulating the infra-red-active gases.
A simple model but in fact very efficient.

The stack model analyzes the long-wave (LW) heat transport through a stack of grids in a vacuum column of 30 km high.
A grid(i) is characterized by the height $z(i)$ and an absorption coefficient $f(i)$, much smaller than 1.
The coefficient $f(i)$ represents the ratio of the cross-section of the wires of the grid(i) with respect to the total surface area of the grid(i).
The layers have given temperatures in degrees Kelvin, $T(i)$ [°K].
We define ftot = $\Sigma f(i)$, it represents the total absorption of the stack, ftotH2O in the water model and ftotCO2 in carbon dioxide model.
The heat flux φ between two grids in W/m² become:

$$\varphi(i \rightarrow j) = f(i,j)*\sigma*(T(i)^4 - T(j)^4) \text{ for } T(i) > T(j) \text{ and } \varphi(j \rightarrow i) = 0 \quad (1)$$

$$\text{With } \sigma = 5.67e\text{-}8 \text{ W/m}^2/°K^4 \quad \text{and} \quad 1/f(i,j) = 1/f(i) + 1/f(j) - 1$$

Relation (1) is the classic Stefan-Boltzmann-Christiansen relation for the one-stream formulation of the heat transport by radiation between the wire of grid(i) with cross-section $f(i)$ and temperature $T(i)$, and the wire of grid(j) with cross-section $f(j)$ and temperature $T(j)$. See **Appendix 1**

Finite element method (FEM) techniques have been applied to generate great numbers of simultaneous algebraic equations describing the heat transport in the atmosphere for both the H2O model and the CO2 model.

It has turned out:

- when the f(i) coefficients are taken proportional to the variable density of water vapor in the atmosphere and the measured values T(i) for the temperature, the LW heat transport in a stack of 50 grids over a height of 11.5 km corresponds to results of main-stream authors of an atmosphere with only traces of infra-red-active water vapor.
  Those comparisons have been made with so-called global K&T diagrams.

- when the variable density of CO2 is used for f(i) and the measured values T(i) for the temperature in a stack of 100 grids over a height of 30 km, the LW heat transport corresponds to results of fundamental physics of an atmosphere with only traces of infra-red-active CO2 gas.
  Comparisons are made with the Happer results **[3]** .

What is called *saturation* of CO2 — for ftotCO2 > 1 (or > 400 ppm in a 30 km high stack) — has been demonstrated transparently with a to zero going "*window*" for non-conventional *overlapping* finite elements.
The phenomenon *saturation* of CO2 is often misinterpreted:
it is less correct to say "*CO2 saturates at 400 ppm*", one should rather say:
"*columns CO2 with height of 30 km become saturated at 400 ppm CO2*".
The detailed atmospheric CO2 density distribution follows from the measured atmospheric temperature distribution of the so-called standard atmosphere, taking into account the pressure decrease with height, defined by the Newton gravity relation, discussed in **Appendix 2.**

The stack model of LW radiation can also include the phenomenon of *thermalization* of CO2. As argued by Pangburn **[2],** carbon dioxide molecules hit by LW from the surface, transfer the absorbed energy , before it has been re-emitted by LW radiation, by means of collision with molecules of the bulk of the atmosphere, including the infra-red active H2O molecules. The exchanged heat has lost its CO2 identity and is re-radiated by the large concentration of H2O molecules at H2O frequencies.

The use of FEM in the stack model

The stack model is not using FEM in the classical way of solving differential equations, but rather using FEM strategies to model the phenomenon by generating a great number of simultaneous algebraic relations describing the LW heat transport, based on the one-stream formulation of equation **(1).**

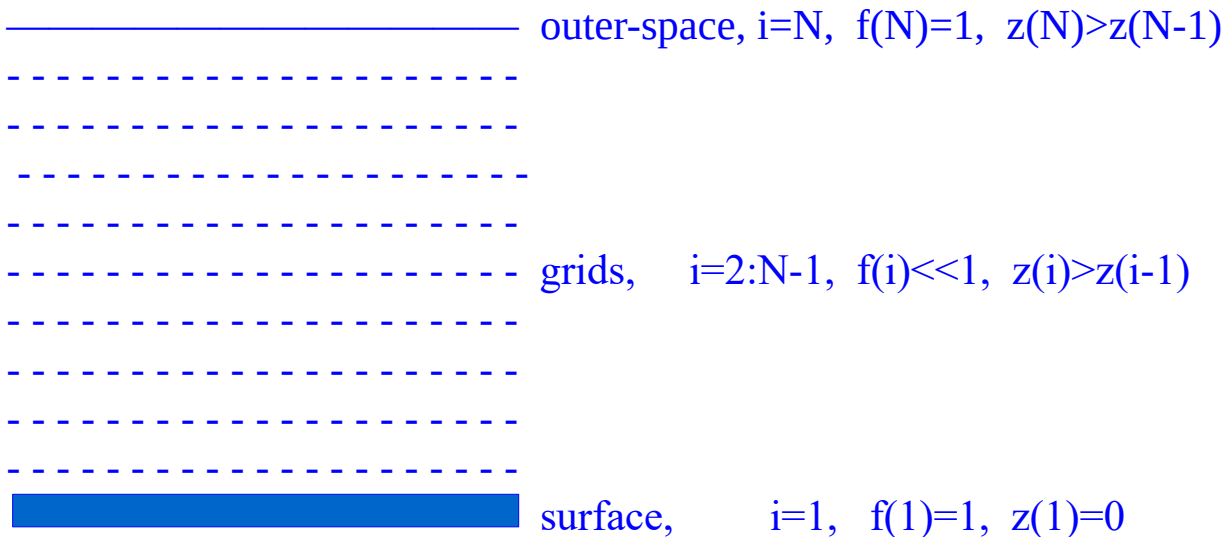We give a short description of this non-conventional FEM approach. We consider in **figure 1** a stack of N-2 grids, with dimensionless small absorption/emission coefficients $f(i) \ll 1$, being the ratio of the cross-section of the wires divided by the total surface.
The coefficients $f(i)$ are assembled in a vector denoted by a bold character **f** of order N, including $f(1)=1$ for the surface and $f(N)=1$ for outer-space.
We define ftot =sum(**f**) - 2, being the sum for the atmospheric grids.
Consider in **figure 1** two layers of black grids with coefficients $f(i)$ and $f(j)$ and absolute temperatures (Kelvin) $T(i)$ and $T(j)$, respectively.

Figure 1 Stack of fine gauze

————————————————— outer-space, i=N, f(N)=1, z(N)>z(N-1)
- - - - - - - - - - - - - - - - - - - -
- - - - - - - - - - - - - - - - - - - -
 -
- - - - - - - - - - - - - - - - - - - -
- grids,   i=2:N-1, f(i)<<1, z(i)>z(i-1)
- - - - - - - - - - - - - - - - - - - -
- - - - - - - - - - - - - - - - - - - -
- - - - - - - - - - - - - - - - - - - -
- - - - - - - - - - - - - - - - - - - -

surface,   i=1,   f(1)=1, z(1)=0

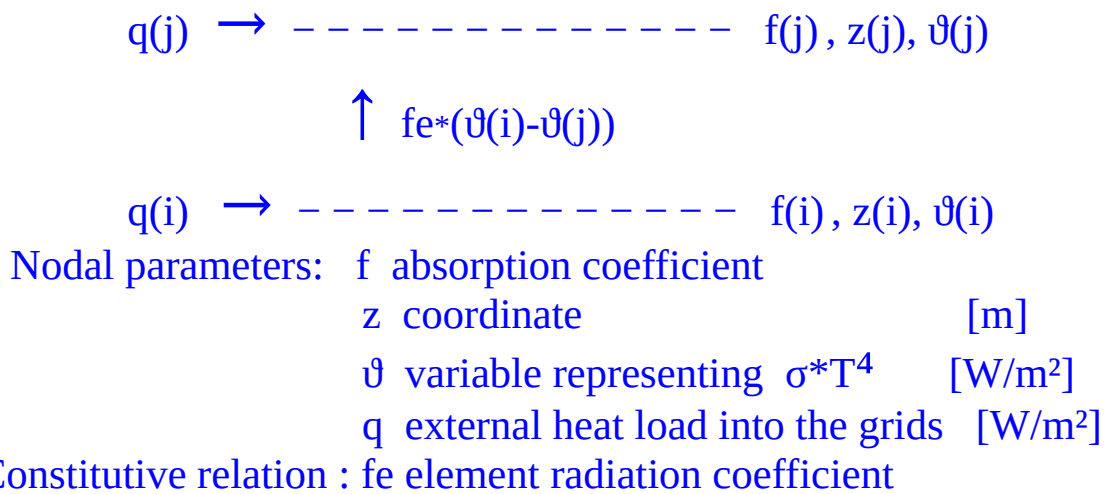The classical Stefan-Boltzmann-Christiansen relation **(1)** with
$$\vartheta = \sigma * T^4 \quad \text{and} \quad 1/fe \ = \ 1/f(i,j) = \ 1/f(i) +1/f(j)-1$$
can be written as:
$$\varphi(i \rightarrow j) = fe*(\vartheta(i)-\vartheta(j)) \qquad\qquad \textbf{(1a)}$$

Figure 2 shows the details of a simple radiation finite element.

Figure 2   Radiation finite element

$q(j)$ $\rightarrow$ – – – – – – – – – – – – – $f(j)$, $z(j)$, $\vartheta(j)$

$\uparrow$ $fe*(\vartheta(i)-\vartheta(j))$

$q(i)$ $\rightarrow$ – – – – – – – – – – – – – $f(i)$, $z(i)$, $\vartheta(i)$

Nodal parameters: f absorption coefficient
z coordinate   [m]
ϑ variable representing $\sigma*T^4$ [W/m²]
q external heat load into the grids   [W/m²]
Constitutive relation : fe element radiation coefficient

By means of a Galerkin-type of variation process, the heat balance of an element(i,j) can be written as:

$$\begin{vmatrix} q(i) \\ q(j) \end{vmatrix} = \begin{vmatrix} fe & -fe \\ -fe & fe \end{vmatrix} \begin{vmatrix} \vartheta(i) \\ \vartheta(j) \end{vmatrix} \qquad (2)$$

Equations **(2)** describe, for given $\vartheta(i)$, $\vartheta(j)$ and fe, the flow of heat by LW radiation between grids i and j and the necessary external heat sources q(i) and q(j), for a balance.

Radiation elements can overlap with each other, as shown in **figure 3.** In a stack of nodes, with increasing height coordinates, any pair of two nodes i and j represents a finite element. For a stack of N nodes there are N(N-1)/2 finite elements with a characteristic heat balance like **(2).**

For an element with **adjacent** nodes, node i and node j = i+1 , the element transfer coefficient is defined by the Christiansen coefficient discussed in **Appendix1:**

$$1/fe = 1/f(i,j) = 1/f(i) +1/f(j) -1 \qquad (3)$$

Figure 3 Illustrative scheme of N(N-1)/2 overlapping
          finite elements (i , j)
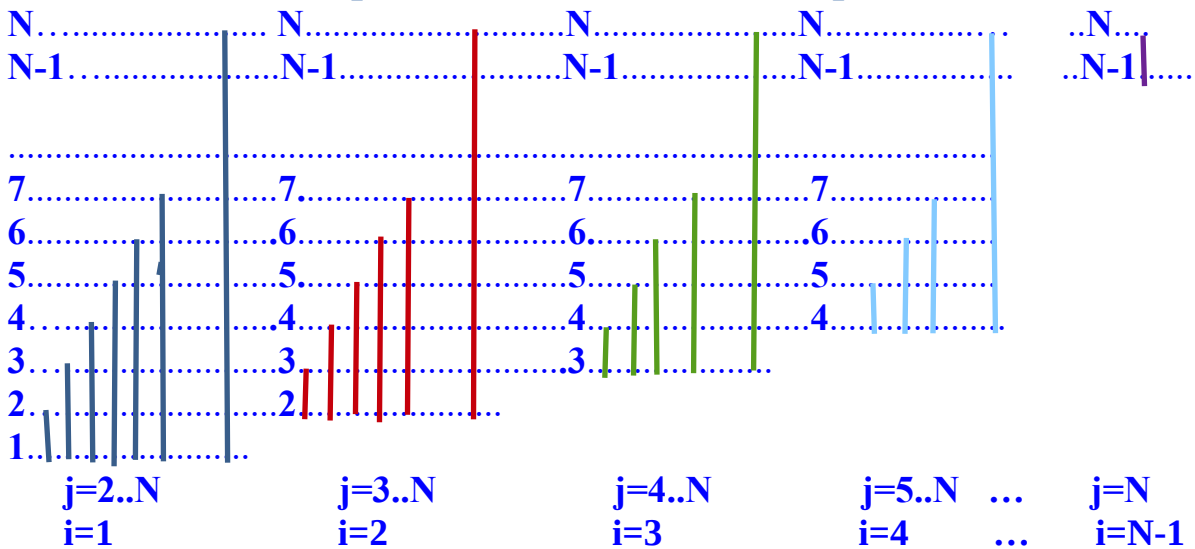
However, when between grid i and grid j of an element other grids of other
elements are present, the transfer of heat by LW radiation between grid i
and grid j of element(i,j) will be hindered and fe of element (i,j) becomes:

$$fe = viewfactor(i,j)*f(i,j) \qquad \textbf{(3a)}$$

In **(3a)** the viewfactor(i,j) takes into account the fact that other grids k are
present between grid i and grid j of an element (i,j).
The viewfactor(i,j) between the nodes i and j of an element is defined by:

$$viewfactor(i,j) = 1 - \sum f(k) \quad \text{for} \quad z(i)<z(k)<z(j) \qquad \textbf{(3b)}$$

The viewfactor(i,j) can be considered as a variable open/closed window
between node i and node j.
Each element(i,j) has a viewfactor(i,j), for N = 100 nodes there are
100*99/2 = 4950 elements(i,j) with 4950 each windows.
(The name window is already used by mainstream authors on the subject
as viewfactor (1,N): between the surface of the planet and outer space.)

The element matrices **(2)** for the N*(N-1)/2 pairs of nodes are assembled
in a system matrix of order NxN, denominated by a bold character **K.**

The characteristic equations of the atmospheric LW radiation become :

$$q = K*\vartheta \tag{4}$$

The reader might appreciate the fact that each of the N nodes in the stack communicates with all the other N-1 nodes.
But the viewfactors limit the communication between nodes depending on the distance between them. Indeed, the stack model with non-conventional overlapping elements represents reality.
For the two infra-red-active molecules of H2O gas and CO2 gas, separate models are used, which in principle do not interfere which each other.
Each model defines an **O**utgoing **LW R**adiation flux, respectively OLR_{H2O} and OLR_{CO2}, depending on the two concentrations of infra-red active gases, water vapor H2O and carbon oxide gas CO2.
They share however common temperatures of atmosphere and surface.
The sum of the two OLR's is equal to the outgoing flux qtoa at the top of atmosphere of the planet equal to the global average of incoming SW radiation from the Sun: $OLR_{H2O} + OLR_{CO2} = qtoa = 240$ W/m²
If for example OLR_{CO2} for some reason would decrease, OLR_{H2O} should increase in order to satisfy the energy balance of the planet.
OLR_{H2O} varies with the surface temperature TsK.
The stack H2O model defines the derivative:

$$dOLR_{H2O}/dTsK = 3.2 \ \ W/m²/^oC. \tag{5}$$

Relation **(5)** and the phenomenon thermalization connect the H2O model with the CO2 model.

IPCC authors claim that for increasing CO2 the planet temperature increases.

However the CO2 stack model shows that for an increasing CO2 concentration beyond 400 ppm the OLR_{CO2} does not decrease anymore: the 30 km high CO2 column at 400 ppm is said to be saturated.

Data for the components of the vector ϑ and the matrix K

Data for these components are shown in **figure 4**: temperature distribution and concentration of water vapor H_2O and of carbon dioxide gas CO_2 over a height of 30 km.

Figure 4

fig 3.1   Standard normalized Temperature,TsK = 288.7209 and exponential decay of water vapor
CO2 distribution based on constant volumetric ppm,
ideal gas law p= rho*R*T, with standard atmosphere for temperature
and with gravitation law of Newton dp/dz = - rho*g
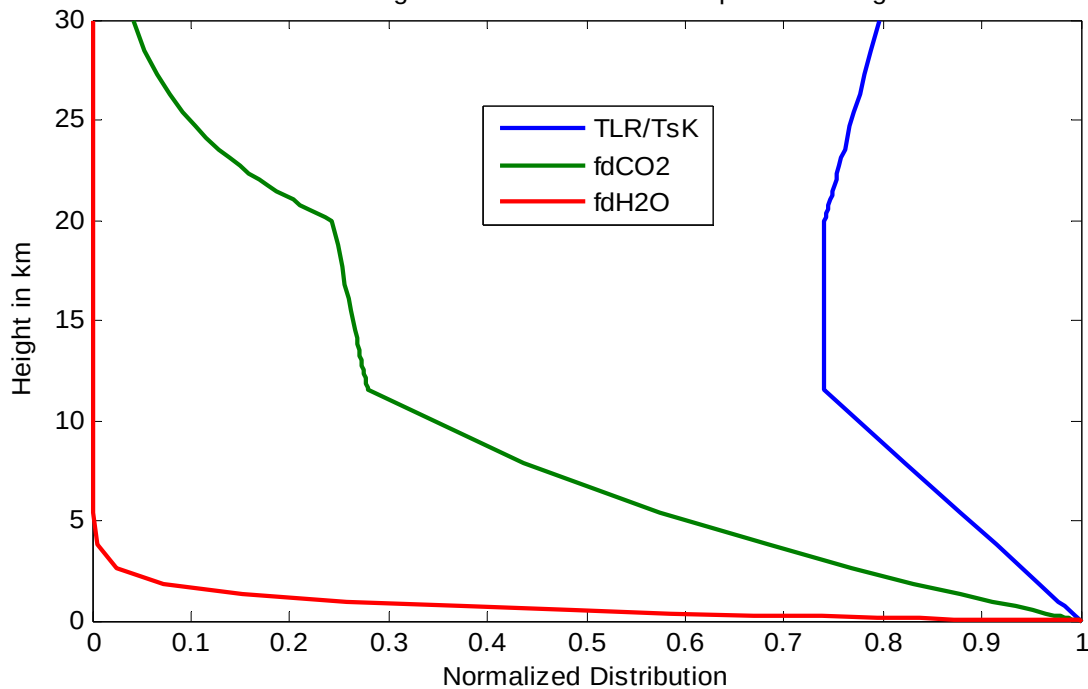
For a height up to 11.5 km the temperature is defined by the surface temperature and the environmental lapse rate, L = dT/dz = -6.5 K/km.
*NB Main-stream papers define **E**nvironmental **L**apse **R**ate ELR = - dT/dz.*
   We prefer to work with a more logical definition L= dT/dz in
   accordance with Taylor expansions.
The temperature distribution is converted to the variables ϑ(i) , assembled in the vector **ϑ.**
With the surface temperature TsK we get the Taylor expansions for TLR(i) and ϑ(i):

$$TLR(i)=TsK + L*z(i) \quad \text{and} \quad ϑ(i) = σ*(TsK + L*z(i))^4 \quad \textbf{(5)}$$

Where z(i) < 11.5 is the vertical coordinate of the grid in km.

For z(i) >11.5 km – for the CO2 analyses -– the temperature distribution follows from **figure 4**, which corresponds to the standard atmosphere. Outer-space temperature is taken as zeroK = 0 or ϑ(N) =0.
In **figure 4** are also depicted the normalized distribution of water vapor and of carbon dioxide gas : fdH2O and fdCO2, respectively.
The normalized H2O distribution is defined heuristically by an exponential drop :   fdH2O(z) = exp(-mH2O*z/height5)
The coefficient mH2O = 7, for a reference height5 of 5 km, is obtained by comparing the results with the main-stream papers on the subject.
The fdCO2 distribution is defined by the pressure and temperature dependent air density, assuming a constant volumetric concentration of CO2 in ppm over the height. See **Appendix 2**

Fractions of H2O and CO2 in the LW terrestrial spectrum
From **figure 5** we can conclude that the fraction of CO2 in the spectrum is 28 +18 =46 W/m² of the total Prevost flux = σ*TsK^4 = 394 W/m² for a surface temperature of TsK = 288.72:
   fractionCO2 = 0.1168   and   fraction H2O= 1-fractionCO2 = 0.8832
Figure 5 from Pangburn blog **[2]**
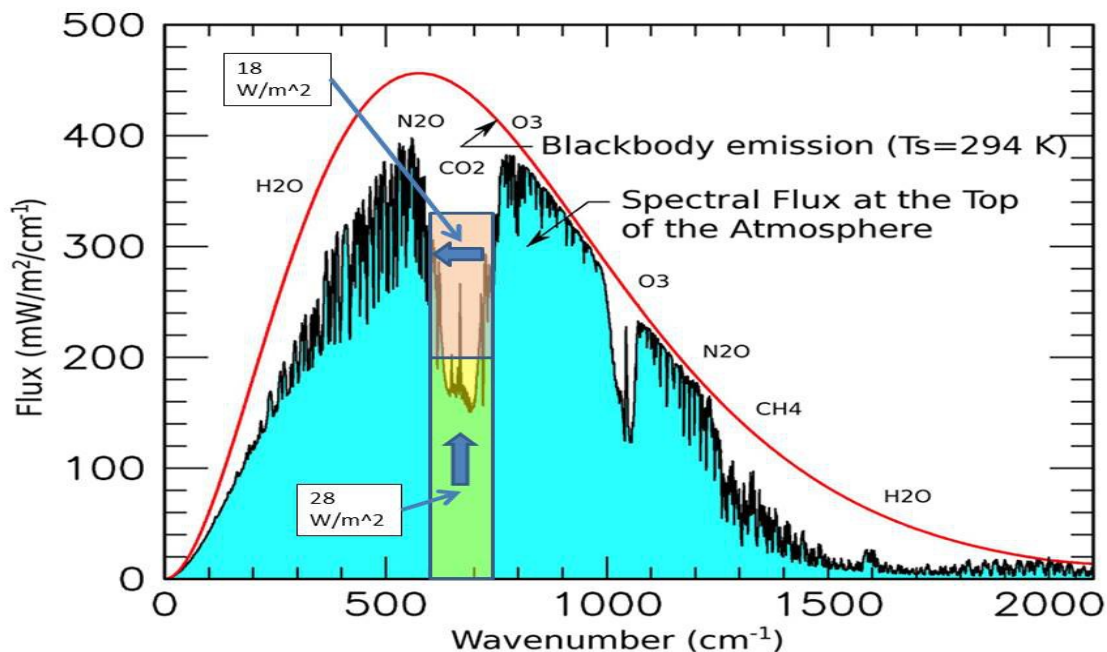
Thermal radiation from below assessed from top-of-atmosphere.
Original graph from NASA
NB Ts =294 K in **figure 4** is a reference value for the red Planck curve.
        Other data are for a temperature of TsK = 288.72.

Results of the stack model for water-vapor.

From **figure 4** we see, for the evacuation of heat through an atmosphere with only water-vapor, a model with a height of 11.5 km is sufficient. The computer program listing (see **Appendix 3**) includes a mesh generator with element sizes based on geometric series: for N=50 nodes of order of 0.1 meter at the surface and of 1.75 km at 11.5 km height.
Figure 6 gives a graphical display of the vector relation **(4): q = K*ϑ.**
It might be useful to repeat in words what the vector relation means:
for a measured temperature distribution given in 50 nodes by a vector of parameters ϑ of order 50 and by multiplication by a radiation matrix K for water-vapor of order 50x50, one obtains a vector q of order 50.

What is the physical interpretation of the components of the vector **q**?
They represent: q(1)   = qsurf = LW surface flux into water-vapor
$\qquad\qquad$ -q(50)   = OLR = outgoing LW radiation of water-vapor
We see in **figure 6** these two components of the vector **q** as function of ftotH2O as well as qtoaH2O =230.12 and:
$\qquad$ qPrevostH2O **=** fractionH2O*****σ*TsK^4 = fractionH2O*ϑ(1)
Figure 6

fig 1.1   Result of H2O analysis with nods =50
nods = 50 mH2O = 7 heightH2Okm = 11.5 LdT/dz = -6.5 degree/km TsK = 288.7209
ftotH2O = 0.7939 fractionH2O = 0.88325 qtoaH2O = 230.176 qPrevostH2O = 347.9998
OLRH2O, qsurfH2O as function of ftotH2O. qwindowH2O = (1-ftotH2O)*qPrevostH2O
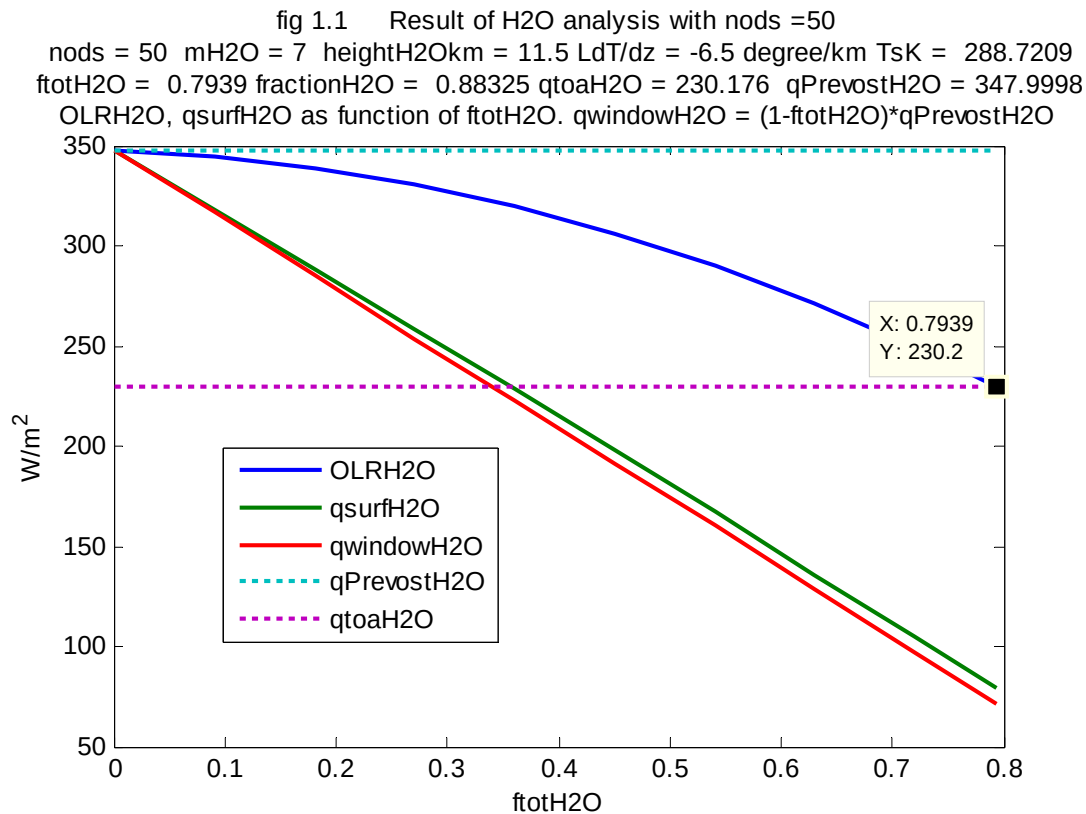
For ftotH2O = 0.7939 a value OLR_{H2O} = 230.18 W/m²: it is the average of the global outgoing LW radiation at H2O frequencies, for which the stack model gives ftotH2O= 0.7939 and a window of (1-ftotH2O) = 0.2061. With OLR_{CO2} = 9.82 from **figure 8**, the average total outgoing heat flux at the top of atmosphere is: OLR_{H2O} + OLR_{CO2} = qtoa = 240 W/m².
The calculated values of the other components of **q** for i = 2 to 49 are given in **figure 7** as function of the height, not as nodal values with dimension W/m² but as distribution in W/m³.
Figure 7

fig 1.2 Result of H2O analysis with nods =50 q = K*theta
Distribution q(2) up to q(49): heat imput by other than LW radiation, such as; convection, SW absorption, CO2 thermalization etc.Total qtot W/m²
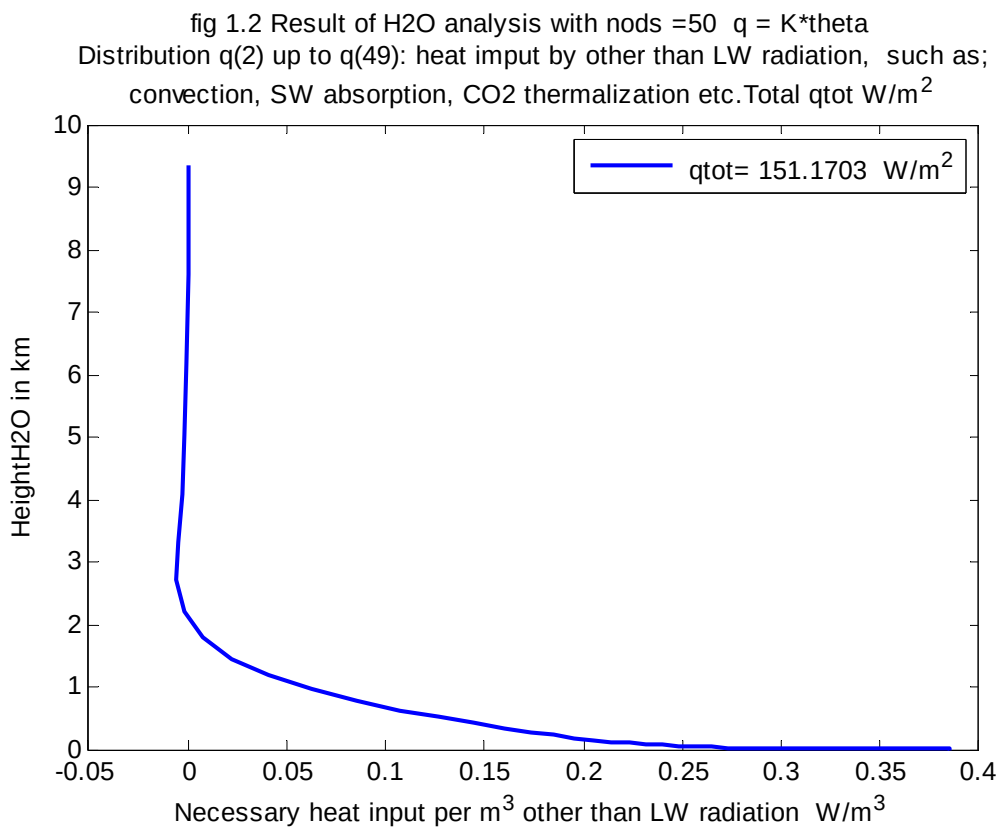
The stack model shows, apart from LW radiation, the additional heat input distribution from the atmosphere with a total value of: 151.17 W/m².
Possible other heat inputs are from:
  – *absorption of incoming SW radiation by aerosols*
  – *convection from the surface of sensible and latent heat,* and
  – *thermalization of CO2 i.e. absorbance in the atmosphere of a part of the infra-red-active gas by LW radiation from the surface but not re-emitted. The heat is given by collisions to the bulk of the atmospheric molecules: 79 % N2, 20 % O2 and the trace water-vapor gas H2O*

The first two contributions are also mentioned by main-stream authors in so-called K&T global energy budgets, the third possible contribution, the thermalization of CO2 introduced in AD 2016 by Pangburn [2], seems to be ignored by authors.

We come back on the phenomenon thermalization further on.

Dependence of OLR on surface temperature

For studies related to the dependence on the ambient temperature of the evacuation of heat from the planet by LW radiation, we need the variation of OLR with the surface temperature TsK.

We repeat equation **(5)** giving the relation between TLR(i) en $\vartheta(i)$:

$$\text{TLR(i)=TsK} + L*z(i) \quad \text{and} \quad \vartheta(i) = \sigma*(\text{TsK} + L*z(i))^\wedge 4 \qquad \textbf{(5)}$$

The variation of $\vartheta(i)$ with the surface temperature is defined by the derivative:

$$d\vartheta(i)/d\text{TsK} = 4\ \sigma*(\text{TsK} + L*z(i))^\wedge 3 = 4*\vartheta(i)\ /\text{TLR(i)}$$

We define the vector **dϑdTsK** consisting of 50 components $d\vartheta(i)/d\text{TsK}$.

The derivative of the vector **q** with TsK becomes the vector:

$$\textbf{dqdTsk} = \textbf{K*d}\boldsymbol{\vartheta}\textbf{dTsK}$$

The derivative of OLR with TsK becomes the minus 50th component:

$$d\text{OLR}_{H2O}/d\text{TsK} = -\ dqd\text{TsK}(50)$$

The stack calculations give: $d\text{OLR}_{H2O}/d\text{TsK}= 3.2342$ [W/m²/°K]

We find a relation for the increase of OLR due to the surface temperature increase. We use the IPCC name for it, forcingOLR:

$$\text{forcingOLR} = (d\text{OLR}_{H2O}/d\text{TsK})*\text{deltaTsK} \qquad \textbf{(6)}$$

Saturation of dense infra-red-active gases

In **figure 8** are given the results of analyses for water-vapor concentrations with ftotH2O >1. We see that the OLR_{H2O} is not decreasing any more. The phenomenon is called saturation and is explained by equation **(3b)**, repeated here:

$$\text{viewfactor}(i,j) = 1 - \sum f(k) \quad \text{for} \quad z(i)<z(k)<z(j) \qquad \textbf{(3b)}$$

For $\sum f(k) > 1$, viewfactor(i,j) would become negative: it is put to zero. For viewfactor(i,j) = 0 there is no LW radiation between the node i and the node j of the element(i,j).

Figure 8

fig 2.1   Result of H2O analysis with nods =50
Watervapor radiation saturation for ftotH2O > 1
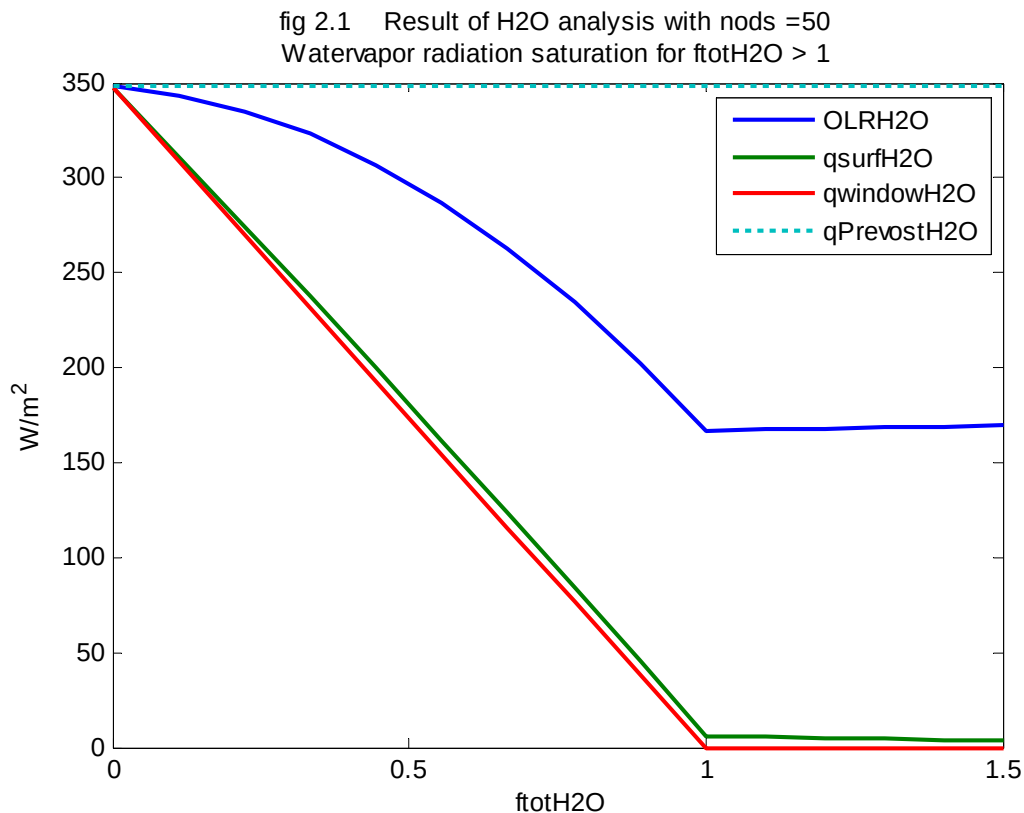
The saturation phenomenon does not appear for water-vapor because the water vapor concentration remains low with ftotH2O < 1.
It is shown here for water-vapor, *for demonstration purposes only,* because it is important for the CO2 analyses further on, with ftotCO2 >1.
Authors very often ignore the CO2 saturation phenomenon, although it is one of the two reasons − *saturation and thermalization of CO2* − for the planet not heating up, as will be shown in the next sections.

Results of the stack model for CO2

The stack model for H2O is a simple one-stream, mono-chromatic model of the evacuation of heat from a planet with only water-vapor.
It turned out from so-called global K&T diagrams to be accurate enough when compared to the results of main-stream authors on the subject, adjusted for the non-physical back-radiation in the two-stream models.
It can also be used for the analysis of CO2 with saturation for values of ftotCO2>1.
For the CO2 analysis we take a model with a height of 30 km with the three 3 temperature zones, according to **figure 4,** 100 nodes with element sizes of 1.4 m at the surface varying to 700 m at the top of atmosphere.
The results of the vector relation **(3)** for CO2, $\mathbf{q = K*\vartheta,}$ are given in **figure 9**, which is equivalent to **figure 6** for the water vapor analysis.
The components of $\mathbf{q}$ and qPrevost represent now:

$$q(1) \quad = \quad qsurf_{CO2} \ = LW \ CO2 \ surface \ flux$$
$$-q(100) \quad = \quad OLR_{CO2} \ = outgoing \ LW \ CO2 \ radiation$$
$$qPrevostCO2 \quad \mathbf{=} \quad \mathbf{fractionCO2*}\sigma*TsK\text{^}4 \ = \ fractionCO2*\vartheta(1).$$

Figure 9   *no thermalization and no saturation*

fig 3.3   Result of CO2 analysis with nods =100   Without thermalisation
heightCO2km =30 ftotCO2 =1 TsK =288.7209 K fractionCO2 =0.11675 qPrevost =46
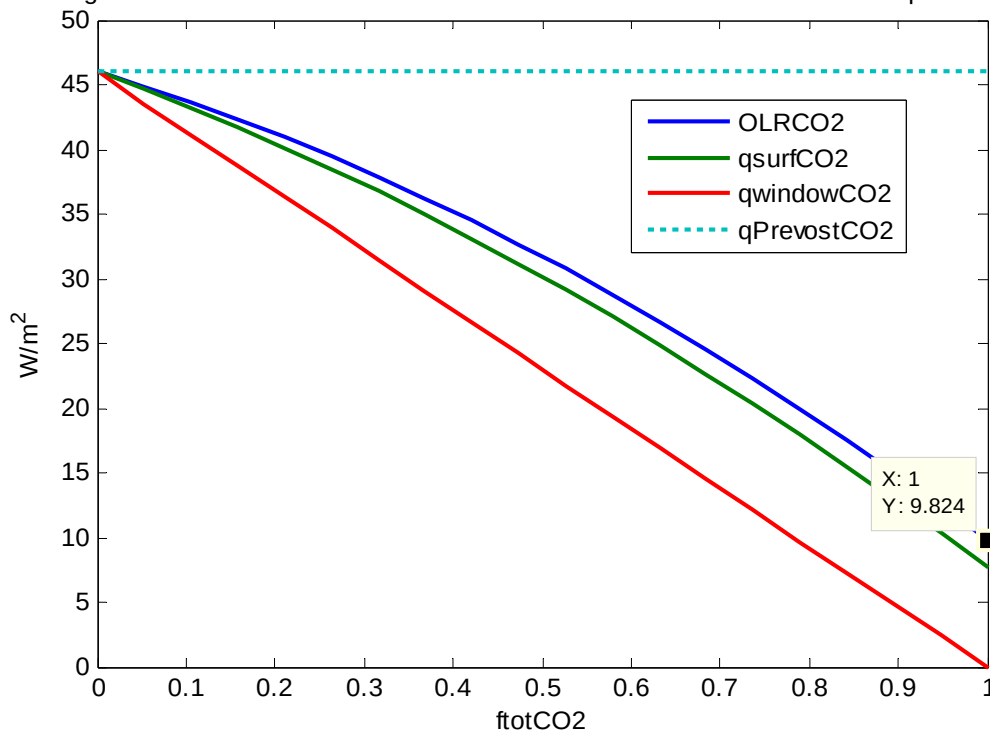
Surface temperature increase due to CO2 increase

In **figure 9** we see a decreasing OLR_{CO2} flux, from $qPrevostCO2 = 46$ W/m² for $ftotCO2 = 0$ towards lower a value 9.824 W/m² for $ftotCO2 = 1$. The decrease $deltaOLR_{CO2}$ as function of OLR_{CO2} becomes:

$$deltaOLR_{CO2} = -(qPrevostCO2 - OLR_{CO2}) \qquad \textbf{(7)}$$

In order to keep the sum $OLR_{H2O} + OLR_{CO2} = qtoa = 240$ W/m² constant, the necessary increase of OLR_{H2O} due to the increase of the surface temperature TsK, called forcingOLR by IPCC, is the opposite:

$$forcingOLR = -deltaOLR_{CO2} = (qPrevostCO2 - OLR_{CO2})$$
(7a)

With the corresponding surface temperature increase from equation **(6)** repeated here:
$$deltaTsK = forcingOLR/ (dOLR_{H2O}/dTsK) \qquad \textbf{(6)}$$

In **figure 10** are given the results from equations **(6)** and **(7)** for CO2 concentrations from $ftotCO2 = 0$ to 1, or 400 ppm.

Figure 10 *no thermalization and no saturation*

fig 3.2 Result of CO2 analysis with nods =100   Without thermalisation
heightCO2km =30 ftotCO2 =1 TsK =288.7209 K fractionCO2 =0.11675 qPrevost =46
dOLRH2OdTsK = 3.2342 deltaTsK = forcingOLR/(dOLRH2OdTsK)   qtherm= 0
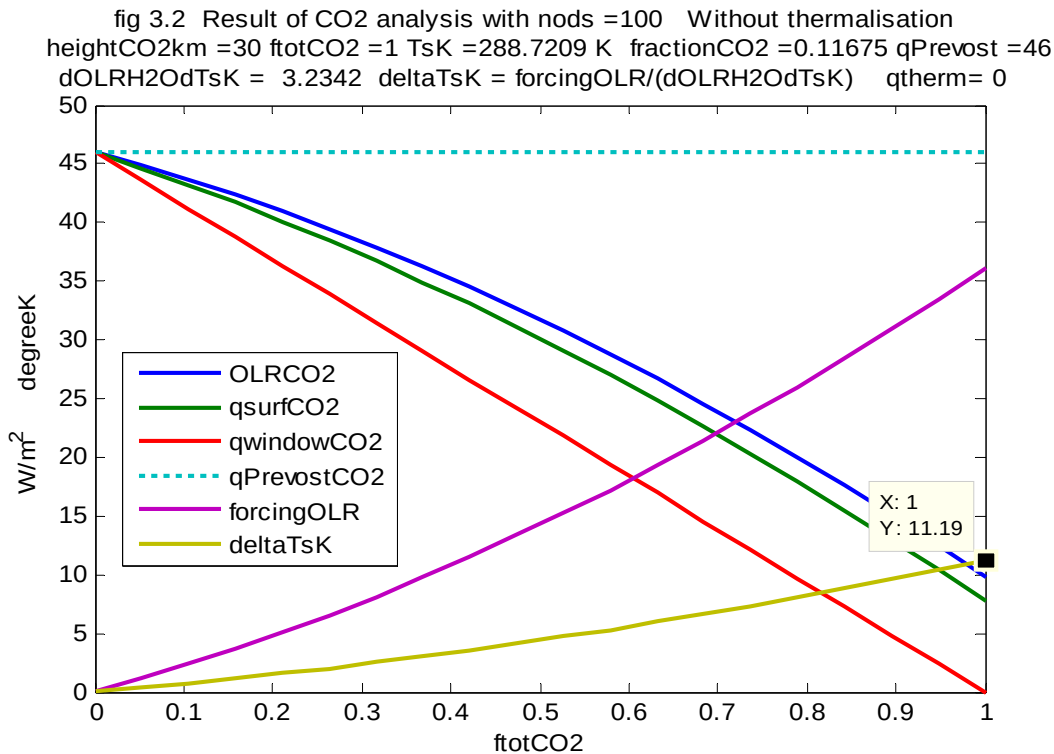
Historical omissions by IPCC authors

In this paper the simple stack models are used to analyze the hindering of LW radiation by infra-red-active gases like water vapor H2O and carbon dioxide CO2, respectively in **figure 6** and **9.**
In **figure 10** are given the results from equations **(5)** and **(6a)** for CO2 concentration increases from ftotCO2 = 0 to 1, or 400 ppm.

With the two-stream Schwarzschild procedures of AD 1906 one can obtain similar numbers but they are seldom shown in detail by IPCC authors.
It is the advantage of the one-stream stack model with efficient graphical display that results can be shown, and not only for one single CO2 value.
The average slope of the deltaTsK curve in **figure 10** between values of ftotCO2 = 0 and =1 becomes 11.17°C.
The so-called sensitivity analysis for double CO2 concentration, for the theory behind **figure 10**, could be represented by this slope.

It were this kind of temperature increases that James Hanssen was referring to in the congressional hearing of 23 June 1988, organized by then senator Al Gore.
It was difficult to look into the future by extrapolation.
However the temperature of the pre-industrial period at 280 ppm was known by measurement and smaller than the corresponding value from **figure 10**: for ftotCO2=0.7= 280 ppm the value for deltaTsK = 6.7 °C.

From the very beginning skeptics have argued that when the theory underlying **figure 10** is not accurate to explain the past, one cannot claim an extrapolation towards higher CO2 concentrations.
Obviously the IPCC slogan ''*science is settled*'' is wrong.
Nevertheless, a value of deltaTsK around 11°C for ftotCO2 = 1 remained to be the temperature from the Schwarzschild analyses, still in use by alarmists and even by some skeptics as starting point for future temperature analyses.

Already in AD 2016 , Pangburn **[2]** has given the reason for the too high numbers in **figure 10**: thermalization of CO2.
We come back to the thermalization phenomenon further on.

Saturation of carbon dioxide

So-called saturation of CO2 is another phenomenon often ignored by IPCC author with the excuse of the slogan " *science is settled* ".

Saturation results for a 30km high CO2 column for concentrations beyond 400 ppm are shown in figure **figure 10a,** up to ftotCO2 = 2 or 800 ppm.

Figure 10a   *with saturation only, no thermaliztion*

fig 3.2 Result of CO2 analysis with nods =100   Without thermalisation
heightCO2km =30 ftotCO2 =2 TsK =288.7209 K fractionCO2 =0.11675 qPrevost =46
dOLRH2OdTsK = 3.2342 deltaTsK = forcingOLR/(dOLRH2OdTsK)   qtherm= 0
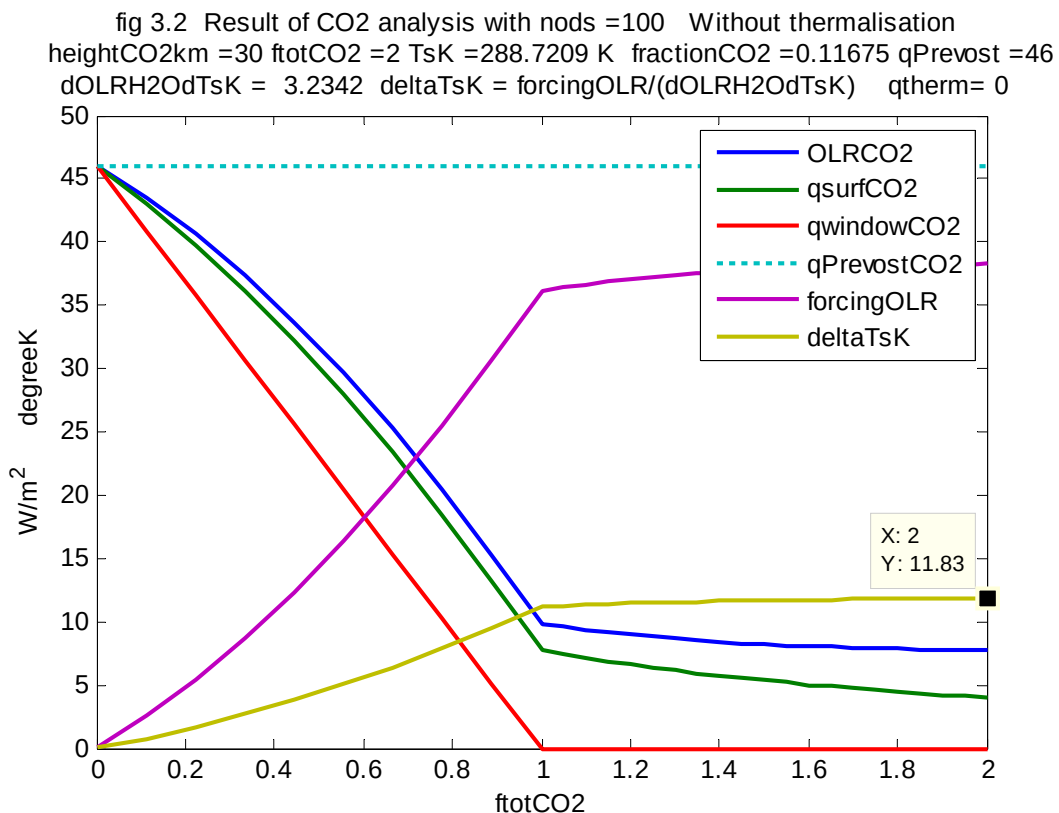
The temperature increase from ftotCO2=1 up to ftotCO2 =2 is small:

$$11.83 - 11.19 = 0.64 \,°C$$

These temperature increases beyond ftotCO2 = 1 are even smaller than the Happer numbers for the region between ftotCO2 = 1 and = 2. [**3**]
We note that for ftotCO2 = 1 the deltaTsK value around 11 °C is too high because thermalization has not been taken into account.

Thermalization of CO2.

The classical Stefan-Boltzmann-Christiansen relation assumes that the information exchange concerning the temperatures between surfaces and thereby exchange of energy is immediate.
There are however different time delays in the process, according to Pangburn **[2].**

Relaxation time:
The time it takes for absorbed energy to be shared with surrounding molecules. It is of the order of a few microseconds.

Decay time:
The time between the absorption process of a molecule CO_2 and the re-emission process, it averages about 1.1 second.

Since the relaxation time is much smaller than the decay time, a CO_2 molecule in the absorption phase collides many times with surrounding 79% nitrogen N_2 molecules and 20% oxygen O_2 molecules, as well as with infra-red-active water-vapor H_2O molecules
The CO_2 molecule loses the surplus energy before it has been built up completely for re-emission!
The CO_2 molecule is said to be thermalized, the surplus energy goes to the bulk of the molecules of the atmosphere.
The exchanged energy has lost its CO_2 identity, the broad band of H_2O frequencies is used for LW radiation to outer space according to **figure 7:** we see that H_2O molecules pick up 151.17 W/m² from the atmospheric sources other than LW radiation from the surface.
See **figure 4** and Pangburn blog **[2]** for further details
In the stack model we have taken into account the thermalization results of Pangburn: for fotCO2 =1 or 400 ppm, a fraction 18 W/m² is thermalized and is radiated to outer space by the broad water vapor H_2O frequency bands according to **figure 7**.
Figure 11 gives results with a thermalization of qtherm = 18 W/m², to be compared with **figure 10,** without thermalization.
For values ftotCO2 = 0 to 1 the distribution of qtherm is assumed to be linear between 0 and 18 W/m².

Figure 11 *with thermalization, saturation not applicable for ftotCO2<1*

fig 3.2therm   Result of CO2 analysis with nods =100   With thermalisation
heightCO2km =30 ftotCO2 =1 TsK =288.7209 K fractionCO2 =0.11675 qPrevost =46
dOLRH2OdTsK = 3.2342 deltaTsK = forcingOLR/(dOLRH2OdTsK)   qtherm= 18
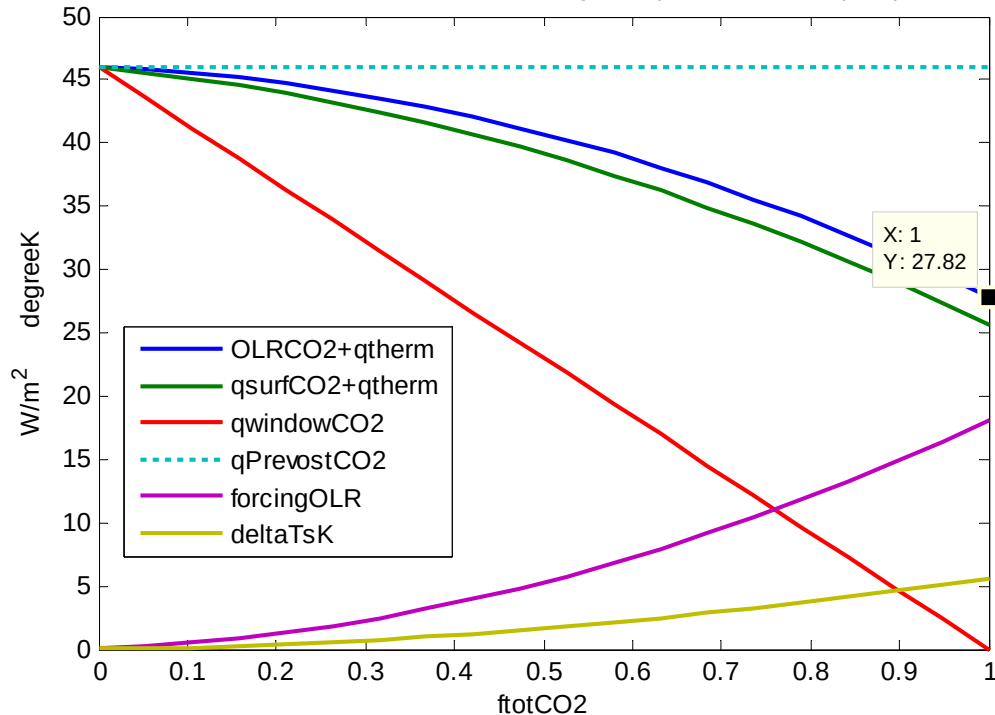
For ftotCO2 = 1:

    q(1) = qsurf$_{CO2}$ = surface flux LW CO2 radiation,
                      including 18 W/m² to be thermalized

 -q(100) = OLR$_{CO2}$ = outgoing LW 9.82 CO2 path + 18 H2O path,
                     total 27.82 W/m²

Pangburn results **[2]** with qtherm =18 W/m² are given for ftotCO2=1 or 400 ppm.

In **figure 11** the thermalization contribution is assumed to be linear between zero for ftotCO2 = 0 and 18 W/m² for ftotCO2=1.

For the pre-industrial value of ftotCO2 =0.7 or 280 ppm the value deltaTsK becomes about 3.00 and for ftotCO2 =1 the value of deltaTsK = 5.603: a difference of 2.6 degree. This difference is presented in **Table1**.

In **figure 11** for ftotCO2<1, only the thermalization is taking into account. For CO2 concentrations beyond the 400 ppm of the year 2015 AD, there is not only thermalization but also saturation, as shown in **figure 12** with the final result of this paper.

Figure 12 *with both thermalization and saturation*

fig 3.2therm   Result of CO2 analysis with nods =100   With thermalisation
heightCO2km =30 ftotCO2 =4 TsK =288.7209 K fractionCO2 =0.11675 qPrevost =46
dOLRH2OdTsK = 3.2342 deltaTsK = forcingOLR/(dOLRH2OdTsK)   qtherm= 18
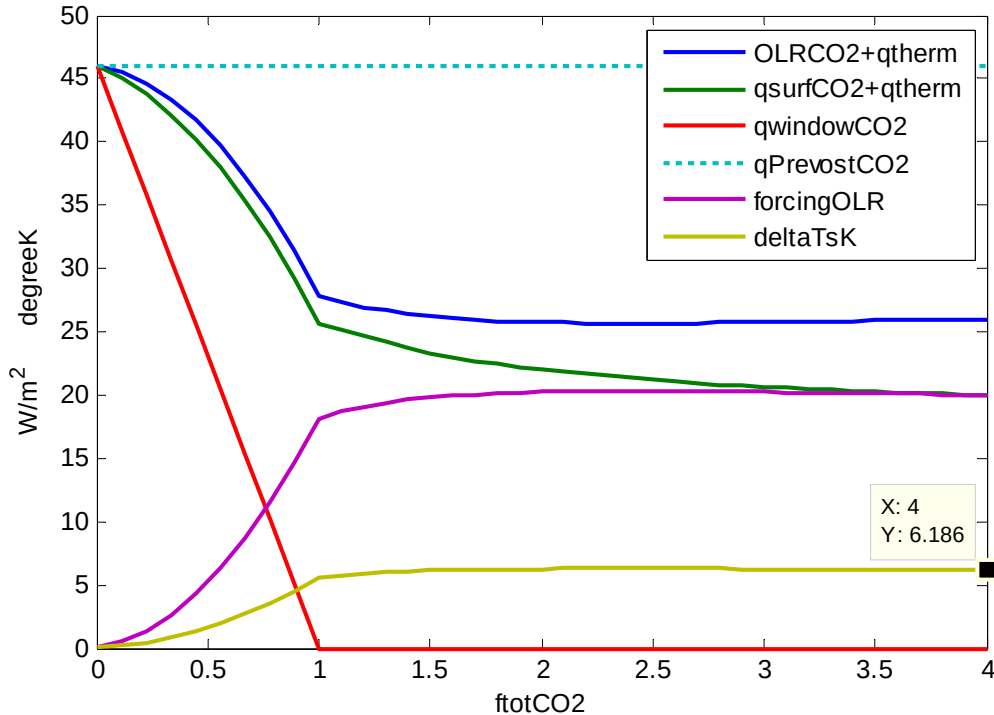
Table 1 summarizes the result of **figure 11** and **figure 12**, giving the temperature increase from ftotCO2 = 0 to ftotCO2 = 4, or 1600 ppm of the year AD 2615, assuming an increase of 2 ppm/yr.

Table 1

Temperature variations in degree C from fotCO2 = 1.

due to thermalization (qtherm = 18 W/m²) and saturation

ftotCO2	ppm	deltaTsK	deltaTsK -5.62	year AD
0.7	280	3.00	**-2.62**	
1	400	5.62	**0**	2015
2	800	6.265	**0.645**	
4	1600	6.186	**0.566**	~ 2615

The difference of - 2.62 °C between the pre-industrial values of 280 ppm or ftot = 0.7 depends on the distribution of the thermalization for ftotCO2<1.

The indicated temperature variations are only due to CO2 concentration variations and for a thermalization of 18 W/m2, reported by Pangburn for CO2 concentration of 400 ppm.

Constant Sun intensity and constant cloud influences are assumed.

Conclusions

The one-stream, stack model for infra-red-active trace gases, based on the classical one-stream Stefan-Boltzmann-Christiansen relation, already validated for the analysis of LW radiation through an atmosphere with water-vapor in so-called K&T diagrams, has now also been applied for the analysis of CO_2 gas and thereby increases of the surface temperature of the planet. The MATLAB .m files are given in **Appendix 3**

The stack model is a simple model.
It deals with the issues of thermalization and saturation of CO_2 in a transparent way.
The two phenomena, thermalization and saturation of CO_2, give rise to small temperature increases, while

- before the year AD 2015 with CO_2 concentration below 400 ppm the thermalization of CO_2 was not published yet by Pangburn

- since the year AD 2016, with higher than 400 ppm CO_2, both thermalization **and** saturation of CO_2 are ignored by IPCC.

Thermalization of CO_2 as reported by Pangburn **[2],** give even lower temperature increases as compared to the saturation only analyses of Happer **[3].**

The infra-red-active gas CO_2 is harmless and non-polluting.

Until nuclear power has been installed back again, we need fossil fuels to generate electricity and for combustion engines of cars, boats and planes.
                    DRILL BABY, DRILL
Burning fossil fuels ─ **fortunately** ─   will increase the concentration of CO_2, also called "greenhouse" gas, which indeed is used in nursery greenhouses to boost the growth of vegetables and flowers.

CO2 is food for plants
 We need more atmospheric CO_2 to feed the growing world population.

Acknowledgment

References

 [1]Reynen
https://principia-scientific.com/wp-content/uploads/2023/04/ SaturationIVnew.pdf.
[2]Pangburn,
https://www.researchgate.net/publication/316885439_Climate_Change_Drivers.
[3]Happer,
 https://www.youtube.com/watch?v=CA1zUW4uOSw
[4]Johnson,
https://computationalblackbody.wordpress.com/

Appendix 1
Stefan-Boltzmann-Christiansen Relation for LW radiation

We consider the LW radiation between surfaces and use the symbols :
- ε coefficient for emission,
  representing the fraction of emitted energy for non-black surfaces compared to the emitted energy of black surfaces. For black surfaces $\varepsilon = 1$, and the energy emitted by a black surface is $q = \sigma_b T^4$ (Stefan-Boltzmann relation, $\sigma_b = 5.67$ e-8 W/m²/ °K⁴)
- α coefficient for absorption,
  representing the fraction of energy absorbed as compared to total energy arriving at the surface.
- ρ coefficient for reflection,
  representing the fraction of energy reflected as compared to total energy arriving at the surface.
- τ coefficient for transparency,
  representing the fraction of energy transmitted through the body as compared to the total energy arriving at the surface.

From the definitions of α , ρ and τ it follows:

$$\alpha + \tau + \rho = 1 \qquad\qquad \textbf{(A1.1)}$$

In **figure A1.1** we show plates 1 and 2 by two vertical lines with two horizontal arrows towards each other, representing energy fluxes:

Figure A1.1

The arrow out of plate 1 represents q_1 the energy in W/m² from plate 1 consisting of the emitted flux $\sigma_1 T_1^4$ out of plate 1 plus the reflection $\rho_1 q_2$ by plate 1 of the flux q_2 from plate 2.

The parameters of the arrow from plate 2 are obtained by cyclic permutation of the subscripts.

We get two simultaneous equations for q_1 and q_2:

$$q_1 = \sigma_1 T_1^4 + \rho_1 q_2 \qquad\qquad \textbf{(A1.2)}$$
$$q_2 = \sigma_2 T_2^4 + \rho_2 q_1$$

In matrix notation:

$$\begin{vmatrix} 1 & -\rho_1 \\ -\rho_2 & 1 \end{vmatrix} \begin{vmatrix} q_1 \\ q_2 \end{vmatrix} = \begin{vmatrix} \sigma_1 T_1^4 \\ \sigma_2 T_2^4 \end{vmatrix} \qquad\qquad \textbf{(A1.2a)}$$

By inverting the matrix we find, with determinant det $= 1 - \rho_1 \rho_2$:

$$\begin{vmatrix} q_1 \\ q_2 \end{vmatrix} = 1/\det \begin{vmatrix} 1 & \rho_1 \\ \rho_2 & 1 \end{vmatrix} \begin{vmatrix} \sigma_1 T_1^4 \\ \sigma_2 T_2^4 \end{vmatrix} \qquad\qquad \textbf{(A1.3)}$$

The algebraic expression for q_1 and q_2 become:

$$q_1 = (\sigma_1 T_1^4 + \rho_1 \sigma_2 T_2^4)/\det \qquad\qquad \textbf{(A1.3a)}$$
$$q_2 = (\sigma_2 T_2^4 + \rho_2 \sigma_1 T_1^4)/\det$$

Kirchhoff and Christiansen relations for non-transparent plates

For $\tau = \tau_1 = \tau_2 = 0$ the relation **(A1.1)** becomes:

$$\rho = 1 - \alpha \qquad\qquad \textbf{(A1.1a)}$$

With $\rho_1 = 1 - \alpha_1$, $\rho_2 = 1 - \alpha_2$, $\sigma_1 = \varepsilon_1 \sigma_b$, $\sigma_2 = \varepsilon_2 \sigma_b$ we find

$$q_{1,2} = q_1 - q_2 = (\alpha_2 \varepsilon_1 \sigma_b T_1^4 - \alpha_1 \varepsilon_2 \sigma_b T_2^4)/\det \qquad\qquad \textbf{(A1.4)}$$

When the plates have the same temperature, $T_1 = T_2$, there is no heat flux between the plates and we find the relation:

$$\alpha_2 \varepsilon_1 - \alpha_1 \varepsilon_2 = 0 \qquad\qquad \text{or} \qquad\qquad \varepsilon_1/\alpha_1 = \varepsilon_2/\alpha_2$$

By a thought process with e.g. plate 1 is black with $\varepsilon_1 = \alpha_1 = 1$ we find that

$$\varepsilon_2/\alpha_2 = 1 \qquad\qquad \text{or} \qquad\qquad \varepsilon_2 = \alpha_2 .$$

Again by a thought process, the value of the emission of the surface of plate 2 does not depend on the surface of plate1, and the conclusion is that emission and absorption coefficients are always equal:

$$\varepsilon = \alpha \qquad\qquad\qquad \textbf{(A1.5)}$$

This is the Kirchhoff relation of 1860!

From equation **(A1.5)** we find :

$$q_{1,2} = \alpha_1 \, \alpha_2 \, \sigma_b (T_1^4 - T_2^4)/\det \qquad\qquad \textbf{(A1.4a)}$$

The determinant, $\det = 1 - \rho_1 \, \rho_2,$ becomes with the relation:

$$\det = 1-(1- \alpha_1)(1- \alpha_2) = \alpha_1 + \alpha_2 - \alpha_1 \, \alpha_2 \qquad\qquad \textbf{(A1.6)}$$

We find: $\alpha_{12} = \alpha_1 \, \alpha_2 / (\alpha_1 + \alpha_2 - \alpha_1 \, \alpha_2) \qquad$ or $\qquad 1/\alpha_{12} = 1/\alpha_1 + 1/\alpha_2 - 1$

Expressed in emission coefficients:

$$q_{1,2} = \sigma_{1,2} (T_1^4 - T_2^4) \qquad\qquad \textbf{(A1.7)}$$
$$\text{with} \quad 1/\sigma_{1,2} = 1/\sigma_1 + 1/\sigma_2 - 1/\sigma_b \qquad \sigma_{1,2} = \varepsilon_{1,2} \, \sigma_b$$
$$1/\varepsilon_{1,2} = 1/\varepsilon_2 + 1/\varepsilon_2 - 1$$

This is the Christiansen relation from 1883.

When both surfaces have emission coefficients $= 1$ then also $\varepsilon_{1,2} = 1$.
When only one surface 1 has an emission coefficient $= 1$ then $\varepsilon_{1,2} = \varepsilon_2 .$
When only one surface 2 has an emission coefficient $= 1$ then $\varepsilon_{1,2} = \varepsilon_1 .$
When the emission coefficients of both plates are $\varepsilon < 1$ then $\varepsilon_{1,2} = \varepsilon/(2 - \varepsilon).$
The Kirchhoff and Christiansen relations (A1.5) and (A1.7) are valid for non-transparent plates ($\tau_1 = \tau_2 = 0$) and are radiation frequency dependent.

The Christiansen relation for transparent plates.

The grids in the stack model are transparent.
Relation **(A1.1a)** is not valid anymore, we have to go back to relation **(A1.1)**:

$$\alpha + \tau + \rho = 1 \qquad\qquad \textbf{(A1.1)}$$

We can introduce parameters $\beta = \alpha + \tau$ and equation **(1)** becomes:

$$\beta + \rho = 1 \qquad \text{and} \qquad \rho = 1 - \beta = 1 - (\alpha + \tau) \qquad \textbf{(A1.1b)}$$

Inserting these reflection coefficients in the relations **(A1.4)** it is sufficient to replace the α by β and we get:

$$q_{1,2} = q_1 - q_2 = (\beta_2\,\varepsilon_1\,\sigma_b T_1{}^4 - \beta_1\,\varepsilon_2\,\sigma_b T_2{}^4)/(\beta_1 + \beta_2 - \beta_1\beta_2) \qquad \textbf{(A1.4b}$$

For the grids in the stack model the transmission coefficients τ are nearly equal 1 and we will use the f coefficients:

$$\tau = 1 - f. \qquad \beta = \alpha + 1 - f = 1 - (f - \alpha) \qquad \rho = 1 - \beta = f - \alpha$$

The transmission through the elements is taken care of by the viewfactors. The transfer between grids in the stack model can be considered as a transfer between plates with $\varepsilon = \alpha$ and the Stefan-Boltzmann-Christiansen for radiation between the grids in the stack model become like equation **(A1.7)**

$$q_{1,2} = \sigma_{1,2}\,(T_1{}^4 - T_2{}^4) \quad \text{with } 1/\sigma_{1,2} = 1/\sigma_1 + f_1/f_2\,(1/\sigma_2 - 1/\sigma_b) \qquad \textbf{(A1.7a)}$$

The relation A1.7a is given by Christiansen for radiation between a smaller convex body inside a greater concave body.
C. Christiansen, " Annalen der Physik und Chemie" , Leipzig, 1883.
This paper is difficult to acces, the reason that the author has given in this appendix a translation of his lecture notes from Delft University of 1955.

In the stack model, we can treat the f coefficients, representing relative surfaces in the same way as emission coefficients with the relation:

$$1/f_{1,2} = 1/f_1 + 1/f_2 - 1 \qquad \textbf{(A1.7b)}$$

When both factors f_1 and f_2 in an element are 1 — in case of a radiation between the surface and outer-space — the coefficient $f_{1,2}$ becomes also 1 although the heat flux is decreased by viewfactor(1,nods) = 1-ftot, and viewfactor(1,nods) = 0 for ftot>1.
When only one surface 1 has an emission coefficient $f_1 = 1$ then $f_{12} = f_2$
When only one surface 2 has an emission coefficient $f_2 = 1$ then $f_{12} = f_1.$
When the two emission coefficients both are $f < 1$ then $f_{12} = f/(2-f)$

For the CO2 analyses the coefficients for ftot =1 or 400 ppm become of the order of 10e-4 and the CO2 analysis can be reduced by one single element from surface to outer-space at 30 km high ,with the reduction by means of the viewfactors.

For the water vapor analysis however, the stack model has shown that the water vapor picks up about 150 W/m² from the atmosphere over a height of only 2 km.
For the water vapor analyses the Christiansen relation is necessary.

Appendix 2

Physical properties of the atmosphere

In **figure A2.1** are given the data, needed for the analysis of the LW heat transport in the atmosphere.

FigureA2.1

fig 3.1   Standard normalized Temperature,TsK = 288.7209 and exponential decay of water vapor
CO2 distribution based on constant volumetric ppm,
ideal gas law p= rho*R*T, with standard atmosphere for temperature
and with gravitation law of Newton dp/dz = - rho*g
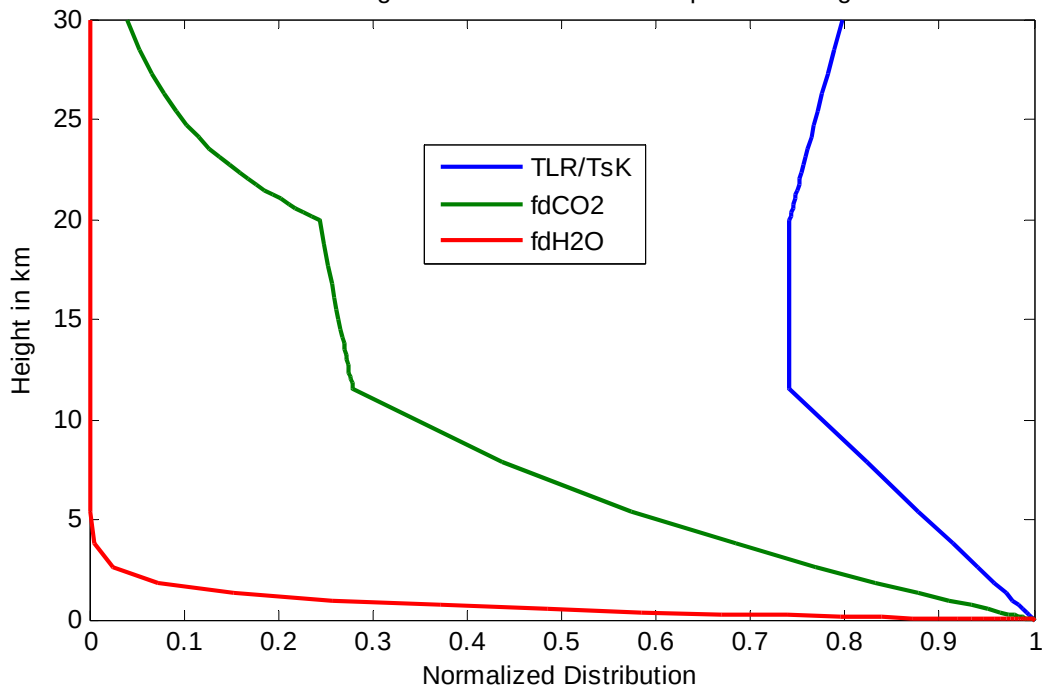
The temperature distribution corresponds to the standard atmosphere with the three zones :

- the lower zone with the Environmental Lapse Rate L_1 = -6.5 °C/km
   up to 11.5 km
- the isotherm middle zone with lapse rate $L_2 = 0$
   between 11.5 km and 20 km
- the upper zone with positive lapse rate L_3 = 1.6 °C/km

From this temperature distribution, using the ideal gas law and the gravity law of Newton we can define the density variation of the air in the atmosphere.

Since the volumetric ppm of CO2 is constant one can define the mass concentration variation of CO2 in the atmosphere.

Dry Adiabatic Lapse Rate, DALR

We consider the **Ideal Gas Law:**

$$p/\rho = RT \quad \text{with} \quad R = c_p - c_v , \quad v = 1/\rho \quad \longrightarrow \quad pv = (c_p - c_v)\, T \quad \text{(A2.1)}$$

p	pressure	[N/m²]
ρ	density	[kg/m³]
v	specific volume	[m³/kg]
T	absolute temperature	[° K]
R	specific gas constant	[Joule/kg/°C]
c_p	specific heat for constant pressure	[Joule/kg/°C]
c_v	specific heat for constant volume	[Joule/kg/°C]

In differential form the ideal gas law can be written as:

$$vdp + pdv = (c_p - c_v)dT \quad \text{(A2.2)}$$

Two equivalent expressions result for the First Law of Thermodynamics:
(a) $dQ = c_p\, dT - vdp$ or   **(b)** $dQ = c_v\, dT + pdv$

dQ external heat supply   [Joule/kg]

The two expressions are equivalent: when subtracted from each other the **dQ** term drops and the differential formulation of the ideal gas law results. When no external heat is present, the First Law **(a)** can be written in adiabatic version with **dQ** = 0:

$$c_p\, dT - vdp = 0 \quad \text{(A2.2a)}$$

Gravitation law of Newton

$$dp/dz = -\rho g \quad [\,N/m^2\,] \quad \longrightarrow \quad dp = -\rho g dz \quad \text{(A2.3)}$$

The combination of **(A2.2a)** and **(A2.3)** gives the dry adiabatic lapse rate:

$$\mathbf{DALR} = dT/dz = -g/c_p \quad \text{(A2.4)}$$

With **g = 9.81 m/sec²** and **c_p = 1000 J/kg/°C** : **DALR = - 9.81 °C/km**

The *measured* lapse rate outside the polar regions, the so-called Environmental Lapse Rate is:

$$ELR = -6.5 \ [^{\circ}C/km] \qquad\qquad (A2.5)$$

In the polar regions the lapse rate is closer to the dry adiabatic lapse rate **DALR**.

It can be concluded that the lapse rate for the bigger part of the planet outside the polar regions up to the equator remains equal to **ELR,** at least in the lower part of the atmosphere up to 11.5 km high.

In the analyses, the standard atmosphere as depicted in **figure A2.1** it has been applied.

NB *Lapse rates are usually defined as $\Gamma = - dT/dz$, resulting in a positive number for the lower atmosphere (see Wikipedia, lapse rates).*
The reason is that in earlier times very often elementary graphical procedures were used in meteorological analyses: negative slopes are not straight forward to use in such elementary graphical procedures.

In computer programming it is more convenient to define a variation of temperature with the Taylor expansion **T= TsK +L*z**, where **TsK** surface temperature in °K and **z** the vertical coordinate and **L = dT/dz**.
And we get **L = - 6.5°C/km** as the Environmental Lapse Rate.

CO2 density variation in the atmosphere

The volumetric concentration of CO2 in ppm on the planet is more or less constant, from the equator to the polar regions and from the surface of the planet up to a height of 30 km and higher.
From the ideal gas law we find:

$$p = \rho * R * T \quad \longrightarrow \quad dp/dz = R*T*d\rho/dz + \rho * R * dT/dz$$

Inserting the gravitation law of Newton;

$$dp/dz = -\rho*g \quad \text{one obtains:} \quad R*T*d\rho/dz = -\rho*(R*dT/dz+g)$$

In case in a section with starting temperature T_1 and lapse rate $L= dT/dz$, which can be positive (zone 3) or negative(zone 1):

$$d\rho /\rho = -(1+g/(R*L))*(L*dz /(T_1+ L*z))$$

Integrating gives:

$$\ln(\rho) = -(1+g/(R*L)) *\ln(T_1+ L*z) +\text{constant}$$

Or, for a region starting at T_1 and lapse rate L the density at a bigger z coordinate with temperature $T =T_1 + L*z$ can be written as:

$$\rho/\rho_1 = (T/T_1)\wedge(-(1+g/(R*L)) \tag{A2.6}$$

When $L = dT/dz =0,$ a constant temperature T results and we obtain;

$$d\rho /\rho = -g/(R*T)*dz$$

Integrating gives: $\ln(\rho) = -(g/(R*T)*z + \text{constant}$

Or for a region starting with ρ_1 and a height z_1 :

$$\rho/\rho_1 = e\wedge(-g/(R*T)*(z -z_1)) \tag{A2.7}$$

The curve for normalized CO2 distribution in **figure A2.11** is obtained by applying for the three zones:
- Equation **(A2.6)** for the nodes in zone 1 with $\rho_1 =1$ and $T_1 =TsK,$ the given surface temperature and lapse rate L_1.
  The last node of zone 1 (nods1) becomes the first node of zone2.

 - Next, equation **(A2.7)** is applied for the nodes in zone 2 with
   $\rho_1 = \rho(nods1)$, calculated in zone1, and the constant temperature
   $T =T(nods1)$

- Next, equation **(1)** is applied for the nodes in zone 3 with
  ρ1 = ρ(nods1+nods2) and
  T(i) = T(nods2) + (znod(i)- znod(nods1+nods2))*L₃

In the curve of **figure A2.1** for the normalized CO2 density distribution, the three zones are clearly recognized.

For the water vapor density distribution an empirical exponential relation is used:

$$H2O(i)=exp(-mH2O*znod(i)/5000) \tag{A2.7}$$

The parameter **mH2O = 7** is defined by comparison of the stack results with K&T diagrams of mainstream authors.

Appendix 3

In this appendix are copied the m.files of the layersjan25.m main MATLAB program.
The user who has difficulties to download these files on a PC with MATLAB program installed, may contact the author for new files jwreynen@gmail.com

```
%plotoption3.m 3


    figure       %fig 3.1 input data for watervapor, CO2 density and temperture
                 %as function of height.

    plot(TLR/TsK,znod/1000,CO2,znod/1000,H2O,znod/1000,'LineWidth',2)
    xtitle1 = ['  Standard normalized Temperature,TsK =  ',num2str(TsK),...
           '  and exponential decay of water vapor'];
    xtitle2 = ['CO2 distribution based on constant volumetric ppm,'];
    xtitle3 = ['ideal gas law p= rho*R*T, with standard atmosphere for
temperature']
    xtitle4 =['and with gravitation law of Newton dp/dz = - rho*g']
    xtitle = ['fig 3.1 ',xtitle1];
    title({xtitle;xtitle2;xtitle3;xtitle4;})
    legend('TLR/TsK', 'fdCO2','fdH2O')
    xlabel('Normalized Distribution')
    ylabel('Height in km')




  xtitle1 = ['fig 3.2  Result of CO2 analysis with nods =',num2str(nods), '
Without thermalisation '];
  xtitle2 = ...
        [  '   heightCO2km =',...
         num2str(heightkmCO2),' ftotCO2 =',num2str(maxftot),...
      ' TsK =', num2str(TsK),' K  fractionCO2 =',num2str(fractionCO2),' qPrevost
=',num2str(qPrevost)];
  xtitle3 = ['dOLRH2OdTsK =  ',num2str(dOLRH2OdTsK),'  deltaTsK =
forcingOLR/(dOLRH2OdTsK)',...
              '   qtherm= 0 ']
  xtitle4 = ['qtherm=  ',num2str(qtherm)];
  figure                                        %without thermalisation
qtherm==0
  plot(fp,OLR,fp,qsurf,fp,qwindow,fp,qPrevostv,':',...
                 fp,forcingOLR,fp,deltaTsK,'LineWidth',2)
  title({xtitle1;xtitle2;xtitle3});
  legend('OLRCO2','qsurfCO2','qwindowCO2','qPrevostCO2','forcingOLR','deltaTsK')
  xlabel('ftotCO2','LineWidth',2)
  ylabel('W/m^2     degreeK','LineWidth',2);

  if qtherm>0
  figure                                        % with thermalisation  qtherm
>0
  plot(fp,OLRtherm,fp,qsurftherm,fp,qwindow,fp,qPrevostv,':',...
                 fp,forcingOLRtherm,fp,deltaTsKtherm,'LineWidth',2)
```

```
   xtitle1th = ['fig 3.2therm   Result of CO2 analysis with nods =',...
                num2str(nods), '   With thermalisation '];
   xtitle3 = ['dOLRH2OdTsK =  ',num2str(dOLRH2OdTsK),...
              '  deltaTsK = forcingOLR/(dOLRH2OdTsK)','   qtherm=
',num2str(qtherm)];
   title({xtitle1th;xtitle2;xtitle3});
   legend('OLRCO2+qtherm','qsurfCO2+qtherm','qwindowCO2',...
          'qPrevostCO2','forcingOLR','deltaTsK')
   xlabel('ftotCO2','LineWidth',2);
   ylabel('W/m^2     degreeK','LineWidth',2);
   end

   figure        %fig 3.3
   plot(fp,OLR,fp,qsurf,fp,qwindow,fp,qPrevostv,':','LineWidth',2)
   xtitle1 = ['fig 3.3    Result of CO2 analysis with nods =',...
              num2str(nods),'   Without thermalisation' ];
   title({xtitle1;xtitle2});
   legend('OLRCO2','qsurfCO2','qwindowCO2','qPrevostCO2');
   xlabel('ftotCO2','LineWidth',2);
   ylabel('W/m^2','LineWidth',2);

   figure     %fig3.4 Heat exchange other than LW radiation with atmosphere
              %as function of height in W/m^3

   plot(qz,znod/1000,'LineWidth',2)

   xtitle1 = ['fig 3.4  Result of CO2 analysis' ];
   xtitle3a = [  ' Distribution of necessay heat deposit by '...
                 'other than LW radiation,   such as;'];
   xtitle3b = [  ' convection, SW absorption etc.Total qtot W/m^2 '];
   title({xtitle1;xtitle2;xtitle3a;xtitle3b});
   str1=  [ ' qtot= ',num2str(qtot),'  W/m^2' ];
   legend(str1);
   xlabel('Necessary heat input per m^3 other than LW radiation  W/m^3');
   ylabel('Height in km')
   endplotoption3.m
%plotoption2.m
figure % fig 2.1
     plot(fp,OLR,fp,qsurf,fp,qwindow,...
                    fp,qPrevostv,':','LineWidth',2)
     xtitle = ['fig 2.1    Result of H2O analysis with nods =',num2str(nods),];
     xtitle1= [' Watervapor radiation saturation for ftotH2O > 1 '];
     title({xtitle;xtitle1})
     legend('OLRH2O','qsurfH2O','qwindowH2O','qPrevostH2O')
                            %when plotting results for watervapor "H2O" is added
to the variable names
     xlabel('  ftotH2O ','LineWidth',2)
     ylabel('W/m^2','LineWidth',2)

%plotoption1.m

xtitle1a = [' nods = ',...
        num2str(nods), '  mH2O = ', num2str(mH2O),'  heightH2Okm = ',...
        num2str(heightkmH2O),' LdT/dz = ', num2str(1000*L) ' degree/km',' TsK =
', num2str(TsK)];

    xtitle1b = [ ' ftotH2O =  ',num2str(maxftot),' fractionH2O =
',num2str(fractionH2O),...
                 ' qtoaH2O = ',num2str(qtoa),'  qPrevostH2O =
',num2str(qPrevost)];
```

```matlab
    xtitle1c = [  'OLRH2O, qsurfH2O as function of ftotH2O. qwindowH2O = (1-
ftotH2O)*qPrevostH2O'];

  xtitleH2O = [' Result of water vapor analysis '];

  figure                      %fig1.1
  plot(fp,OLR,fp,qsurf,fp,qwindow,fp,qPrevostv,':',fp,qtoav,':','LineWidth',2)
  xtitle = ['fig 1.1     Result of H2O analysis with nods =',num2str(nods),];
  title({xtitle;xtitle1a;xtitle1b;xtitle1c;})
  legend('OLRH2O','qsurfH2O','qwindowH2O','qPrevostH2O','qtoaH2O')
  xlabel('ftotH2O')
  ylabel('W/m^2')


  figure     %fig 1.2

  plot(qz,znod/1000,'LineWidth',2)   %distribution of heat deposits due to the
imposed L
  xtitle = ['fig 1.2 Result of H2O analysis with nods =',num2str(nods), '  q =
K*theta'];
  xtitle1a = [  ' Distribution q(2) up to q(49): heat imput by other than LW
radiation,  such as;'];
  xtitle1b = [  ' convection, SW absorption, CO2 thermalization etc.Total qtot
W/m^2 '];
  title({xtitle;xtitle1a;xtitle1b;})
  str1=  [ ' qtot= ',num2str(qtot),'  W/m^2' ];
  legend(str1)
  xlabel('Necessary heat input per m^3 other than LW radiation  W/m^3')
  ylabel('HeightH2O in km')


%meshCO2.m
%lower atmosphere;
                                  %Tnodg and znodg are defined in main program;
                                  %three regions
 TLR =zeros(nods,1);
 H2O =zeros(nods,1);
 CO2 =zeros(nods,1);
 znod =zeros(nods,1);
 L1 = L;                        %L = dT/dz environmental lapse rate -6.5 degree/km
 L2=0;
 L3=(Tnodg(4)-Tnodg(3))/(znodg(4)-znodg(3))
 qtoaz(1)=qtoa;
 zeroK=0;              % outer spacetemperture is taken as 2 degree Kelvin

%continued mesh at given znodg
    znod(1)=0;
    dz=1
    for i=2:nods
    znod(i)=znod(i-1)+dz;
    dz=dz*1.1
    end
    ratio=znod(nods)/znodg(4);
    znod=ratio*znod;
    znod1=znod(1)
    znodnods=znod(nods)

    H2O(1)=1;
    CO2(1)=1;
    CO21=1;
```

```
    TLR(1)=TsK;
    znodim1=0;
    for i=2:nods;
        znodi=znod(i);
        if znodi<znodg(2)+1
        TLR(i)=TsK + L1*znodi;
        CO2(i)=CO21*(TLR(i)/TsK)^(-(1+ g/(R*L1)));
        CO22=CO2(i);
        else
            TLR(i) =  Tnodg(2);            %constant temperature
            CO2(i) =  CO22*exp(-(znodi-znodg(2))/(R*Tnodg(2)));
            CO23=CO2(i);
        end

        if znodi>znodg(3)-1
          TLR(i)=Tnodg(3) + L3*(znodi-znodg(3));
          CO2(i)=CO23*(TLR(i)/Tnodg(3))^(-(1+ g/(R*L3)));
          CO24=CO2(i);
        end
      H2O(i)=exp(-mH2O*znodi/5000);
     znodim1=znodi;
    end
   CO21
   CO22
   CO23
   CO24
   %plotdistribution

for i=1:nods
   thetai=sigma*TLR(i)^4;
   theta(i)=thetai;
   dthetadT(i)=4*thetai/TLR(i);
end
theta(nods)=zeroK;
dzv =diff(znod);
dzv1=dzv(1);
disp ('dzv1: element size in meter at bottom znod=0')
dzv1
dzvnodsm1 =dzv(nods-1)/1000;
disp('dzvnods: element size in km at top of atmosphere 30 km ')
dzvnodsm1
 for i=2:nods-1
     fd(i)= dzv(i)*CO2(i);
 end
fd(1)=0;
fd(nods)=0;
totfd=sum(fd);
fd=fd/totfd;
%makeviewfactor.m
viewfactor = ones(nods);

  for i=1:nods-2
    for j=i+2:nods
        viewfactorij=viewfactor(i,j);
          for k = (i+1):(j-1)
              if k>i
                if k<j
                 viewfactorij=viewfactorij-f(k);
                end
              end
```

```
            end
           if viewfactorij<0
             viewfactorij=0;
           end

       viewfactor(i,j)= viewfactorij;
       viewfactor(j,i)= viewfactorij;   %not needed because i always < j

    end
  end
%makeqz
qz=zeros(nods,1);
for i=2:nods-1
    qz(i)= q(i)*2/(znod(i+1)-znod(i-1));
end
qz(1)=qz(2);
qz(nods)= qz(nods-1);
qtot=sum(q)-q(1)-q(nods);
 %makeK
K = zeros(nods);
for i=1:nods-1
 fi=f(i);
 for j=i+1:nods
 fj=f(j);
 %Christiansen
 factorij=viewfactor(i,j);
 aux=fi*fj/(fi+fj-fi*fj);
 fe=aux*factorij;

 K(i,i)=  K(i,i) +fe;
 K(i,j)=  K(i,j) -fe;
 K(j,i)=  K(j,i) -fe;
 K(j,j)=  K(j,j)+ fe;
 end
end
```

```
 %layers20jan2025.m

%      Various analyses can be carried out by the MATLAB program:
%
%  option(1)
%      Calculation of watervapor heat fluxes q = K*theta with theta =
sigma*(TsK+L*z)^4,
%      where TsK+L*z is the air temperature distribution, with lapse rate
L=dT/dz.
%      NB Environmental Lapse Rate is defined as ELR = -dT/dz.
%        For more transparent programming the definition L = dT/dz is more
logical
%      The analyses are based on the Stefan-Boltzmann-Christiansen relation from
1883
%      for the one-stream heat transfer between two plates with different surface
conditions
%      The calculated fluxes q are:
%          q(1): the first component is the LW surface flux, including the flux
%                through the atmospheric window, directly towards outer space
%      -q(nods): the last component represents the outgoing LW radiation: OLR
%
%      The remaining components are the necessary heat deposits by mechanisms
```

```
%      other than LW radiation to compensate for the loss of heat due to LW
%      emission by the traces of IR-active gases!
%      The air temperature distribution is given by the lapse rate L =dT/dz and
%      the surface temperature.
%      Figures are given of LW surface flux and OLR (outgoing LW radiation)as
function of ftot.
%      The difference between OLR and qsurf is the heat deposited in the
%      atmosphere by convection of active and latent heat and the heat
%      due to absorption of SW incoming radiation.
%
%      option 2
%      A demonstration of the saturation phenomenon with hypothetical water
%      vapor with ftot >1.
%
%      option (3)
%       The saturation phenomenon of CO2 in the atmosphere is demonstrated.
%        CO2 thermalization of 18 Watt/m2 accordingto Pangburn is taken into
account
%       CONCLUSION: CO2 is not warming the atmosphere
%
%   Make a choise in the following MATLAB program

option = 4              %With this value for option the loop is entered with three
possible options

while option >0          % outer loop N
clear                    %loop over the total program, until option =0 or > 3
disp('           LIST OF OPTIONS')
disp('option == 1:    Watervapor analysis')
disp('option == 2:    Demonstration of non-existing saturation of Watervapor')
disp('option == 3:    CO2 sensitivity analysis with saturation and
thermalization ')
disp('   ')
iter=1;                  %in the water model iterations are carried out to find a
solution
                         %for OLRH2O =240-OLRCO2.
while iter>0
option=input('Make a choice for an option, finish with 0:')
  if option < 4
    if option>-1
        iter=0;
    end
  end
     if iter==2
     iter=0;
     option=0;
     end
     if iter == 1
     disp('Option should be between 0 and 4')
     disp('Hit any key and try once more')
     pause
     iter=iter+1
     end
end
disp('input for options finished')

R  = 287.058;          %= cp-cv ideal gas  constant per J/kg
cp = 1000;             % specific heat of air at constant pressure J/kg
g  = 9.81;             % gravity acceleration m/s^2
TsC = 15.7209;         % surface temperature in C
```

```
TsK = TsC+273;          % surface temperature in K
sigma=5.67*10^-8;      % Stefan-Boltzmann constant  W/m^2/K^4
                        % the influence of the lapse rate,
                        % see appendix  in [1] for a derivation from
                        %the ideal gas law and Newtons law for gravitation.
L  = -6.5/1000;        % LdT/dz environmental lapse rate -6.5 K/km
                        %in meteorpology lapse rates are taken positive
                        % because of the graphical procedures in that field.
                        %in the computer software it is better to use the
                        %derivative  as the slope Lapse rate are equal to
                        % L= dT/dz = -6.5 C/km and converted to C/m.
mH2O = 7;               % coefficient for the exponential distribution of water
vapor
                        % users are invited to play with it
                        % mH2O =7 turned out to give results corresponding to
                        % experimental results of Miskolczi publications
                        % see ref 7


qtoa  = 230.176;        %water vapor  flux at top of atmosphere according to
NASA, W/m^2
                        %240 - 9.824 (= CO2qtoa) from runs with the present
software
                        % used in plots for finding reference point, see K&T
                        % diagrams in reference [1,2,3


qPrevost0 = sigma*TsK^4; %the heat flux to outer space without infra-red-actif
gas
                        %like water vapor or CO2 gas.
                        % theta =sigma*T^4 as heat flux or as temperature
parameter
fractionCO2 =46/qPrevost0;  %Pangburn figure from NASA for TsK =294 corrected to
46 see [1]
fractionH2O=1-fractionCO2;
                        %the CO2 notch represent a fraction fractionCO2 of the
total spectrum
                        %this value is taken to show the saturation phenomenon
                        %water vapor, water droplets water and other particles

znodg =zeros(4,1);      %data temperage distibution up to 30 km for CO2
analysis
Tnodg=zeros(4,1);       %standard atmosphere
znodg(1)=0;
znodg(2)=11500;
znodg(3)=20000;
znodg(4)=30000
Tnodg(1)= TsK;
Tnodg(2)= Tnodg(1)+L*(znodg(2)-znodg(1));
Tnodg(3)= Tnodg(2);
Tnodg(4)= 230;


if option < 3           % the water analyses in option 1 and 2, CO2 analysis
                        % for option = 3.

if option ==1            %basic water vapor H2O analysis

givenftot = 0.81;        %this value is calculated by earler runs of the
stackmodel
                         %and based on the general accepted average
                         %outgoing LW flux:OLR=240 W/m2
```

```
nabsorb = 10;                     %the value nabsorb control de output figures.
                                  % it are the number of analyses  as function of
ftotH2o
end
if option == 2          %to show the saturation phenomenon in watervapor
    givenftot=1.5;      %for ftotH2O >1 also water vapor in the air saturates
for heat absorption
                        % 10 analyse between ftotH2O =0 to 1.
    nabsorb1 = 10
    nabsorb  = 15;          % 20 -10 analyses for ftotCo2 =1 to 1.5
end

fp    = zeros(nabsorb,1); %emissio/absorption coefficients values for which
analyses are carried out
                         %graphical results

if option ==1             %nabsorb values to be defined(values for which
analyses are
                          %carried out for water vapor analyses
ftot =givenftot;          %for option 1 and 2 it is also called ftotH2O
fmin=0.001;
fmax = ftot;
 for jf=1:nabsorb
     fp(jf)= fmin+(jf-1)*(fmax-fmin)/(nabsorb-1);
  end
end

if option == 2         %special watervapor analyses to show that for fotH2O >1
                       %saturation occurs also for watervapor.
ftot =givenftot;
fmin=0.001;
fmax = 1;
for jf=1:nabsorb1
     fp(jf)= fmin+(jf-1)*(fmax-fmin)/(nabsorb1-1);
end
fp(nabsorb1)=1;
 fmin=1;
 fmax=givenftot;
  for jf=nabsorb1:nabsorb
        fp(jf)= fmin +(jf-nabsorb1)*(fmax-fmin)/(nabsorb-nabsorb1);
  end
  fp(nabsorb)=givenftot;

end          %end of opton 2
                                  %nabsorb values to be defined(values for which
                                  %analyses are carried out for water vapor

qsurf   = zeros(nabsorb,1);    %LW surface flux, from surface into atmosphere,
qwindow = zeros(nabsorb,1);     %it includes qabsorb and qwindow
OLR     = zeros(nabsorb,1);      %Outgoing Longwave Radiation
qz      = zeros(nabsorb,1);      %heat input other than LW surface radiation
qtoav   = zeros(nabsorb,1);    %make a vector of qtoa and Prevost
qPrevostv =zeros(nabsorb,1);   %for plotting purposesqabsorb  =
zeros(nabsorb,1);
nods = 50;
qzv     = zeros(nabsorb,nods);
thetav  = zeros(nabsorb,nods);
tempv   =  zeros(nabsorb,nods);

             % nodes in the atmosphere for water vapor analysis,
```

```
                              % including surface node=1 and outer space node = nods

heightkmH2O=11.5;       % default height of H2O model in km, can be changed
                        %for water vapor 7km would be sufficient, for CO2 we
                        %use 25


toakm    = heightkmH2O;
heightH2O= heightkmH2O*1000;  % convert data from km into meters

%mesh generation parameters
ratioatm =1.23 ;             % ratio for the geometrical series of distribution of
nods.
                             % for the distribution of IR-active trace gases.

 znod=zeros(nods,1);   %nodal coordinates for watervapor analysis,
                       %ground level at znod=0
 znod(1) = 0;
 for i=2:nods
    znod(i)=znod(i-1)+ratioatm^(i-1);
 end
 ratio=heightH2O/znod(nods);
 znod =ratio*znod;  %coordinates are now between 0 and heightH2O

dzv =diff(znod);        %diff Matlab function to make a vector of differences
dzv;                    %in fact the element sizes
dzv1=dzv(1);
disp ('dzv1: element size in meter at bottom znod=0')
dzv1
dzvnods =dzv(nods-2)/1000;
disp('dzvnods: element size in km at top of atmosphere 11.5 km ')
dzvnods
TLR=zeros(nods,1);                    %temperature distribution in Kelvin
theta=zeros(nods,1);                  %alternative variable for temperature
                                      %theta(i)=sigma*TLR(i)^4
dthetadTsK=zeros(nods,1);             %derivatalternative variable for temperature
for i=1:nods-1
    znodi=znod(i);
    TLRi=TsK+L*znodi;
    TLR(i)=TLRi;                       % temperature distribution  for TsK and L
    theta(i)=sigma*TLRi^4;             % theta are convenient parameters for
temperature
    dthetadTsK(i)=4*theta(i)/TLRi;   %to be used for dOLR/dTsK
end
dthetadTsK;
qPrevost=fractionH2O*theta(1);   % Prevost surface flux
TLR0=TLR;
zeroK= 0 %2;                 % outer space taken as zero,
                            %can also be taken as 2 degree K,
                            %no difference in answers!
theta(nods)=zeroK;               % sigma*zeroK^4;
znod(nods)=znod(nods-1);


f=zeros(nods,1);
f(1)=1;   %fractionH2O;    %emission/absorption coeeficients  of the surface
f(nods)=1;        %outer space f

% distribution of absorption at nods = znod(i)
% exponential decay of absorption with heightH2O defined by mH2O
% for mH2O=0 homogeneous distribution, mH2O=7 gave the best comparison with the
```

```matlab
% experimental results of K&T diagrams
% distribution of CO2 proportional to density

for i=2:nods-1
    znodi=znod(i);
    zref=5000;
    H2Oratio=exp(-mH2O*znodi/zref);
    fd(i)=dzv(i-1)*H2Oratio;
end

fd(1)   =0;                        % at node =1 (surface) and at node =nods
(outerspace)
fd(nods)=0;
totfd   =sum(fd);                  %normalized distribution of absorption, total
normalized = 1
fd=fd/totfd;

f(1)=1;                            %fractionH2O taking take care of later
dzv =diff(znod);        %diff Matlab function to make a vector of differences
dzv1=dzv(1);
dzvnods=dzv(nods-1);

fd(1)  = 0;                        % at node =1 (surface) and at node =nods
(outerspace)
fd(nods)= 0;                       % make fd=0 for both H2O and CO2
                                   %normalized distribution of absorption, total
                                   %normalized = 1
totfd=sum(fd);
fd=fd/totfd;
for i=1:nods
    theta(i)=sigma*TLR(i)^4;   %sigma*T^4
end
totfd=sum(fd);
fd=fd/totfd;


jf=0;
mismatch=0;                        %mismatch is difference with target in iteration
dfj=0;
qPrevost=fractionH2O*sigma*TsK^4;   % Prevost surface flux without infra-red-
active gases

jf=0;
while jf<nabsorb                   % loop through water absorption levels for
options 1, 2, 3
  jf=jf+1;
  qtoav(jf)=qtoa;                  %for plotting a horizontal line=qtoa
  qPrevostv(jf)=qPrevost;          %for plotting a horizontal line = qPrevost
  f=zeros(nods,1);
  fpjf=fp(jf);
  f=fd*fpjf;
  f(1)= 1;                         %surface coefficient f(1)==1
  f(nods)=1;                       % outer space f(nods)=1
 qwindow(jf)= (1-fp(jf))*qPrevost;
  if qwindow(jf) < 0
      qwindow(jf)=0;
  end

    makeviewfactor
    makeK                 %K matrix
```

```matlab
        K=fractionH2O*K;    %in fact fractionH2O is applied to theta

      theta(nods)=zeroK;  %outer space temperatrure at zeroK, just to be sure  in
a
                            %previous plotting option the value is set to
                            %theta(nods-1) for plotting reasons
      q=K*theta;            % q is includes contribution from mechanisms other than
radiation
                            %except first term = qsurf and last term=-OLR
      qwindowjf=(1-fp(jf))*fractionH2O*theta(1);
      if qwindowjf <0
          qwindowjf=0;
      end
      qwindow(jf)=qwindowjf;
      qsurf(jf)=q(1);       %outgoing radiation due to LW surface flux,
                            %including flux through window: qwindow(jf)
      OLR(jf)=-q(nods);     % total outgoing including mechanisms
                            %other than surface radiation

    if option==1
                            %correct for plotting in a non-homogeneous mesh, qinto
and qout are
                            %nodal values, W/m^2, to be converted to volumetric
values W/m^3.
                            %The difference is due to mechanisms other than LW
radiation
        makeqz              % deposit mechanisms other than LW radiation from
surface

        qzv(jf,:)= qz;    %z-distribution of LW for different absorbtions ftot
      end

    OLR(jf) = -q(nods);
    OLRpr=qPrevost;
    if jf >nabsorb-2
      OLRpr=OLR(jf-1);
      dOLRdftot =(OLR(jf)-OLRpr)/(fp(jf)-fp(jf-1));
    end
    qsurf(jf) = q(1);
    qwindowjf=qwindow(jf);

 if jf<nabsorb
     iter=0;
 end
 if option== 1
   if jf==nabsorb
   disp('Iterations to obtain the ftot value for OLR =qtoa =  240  -CO2
contribution ')
     iter =iter+1
     OLRjf=OLR(jf);
     mismatch=OLRjf-qtoa;
     dfj= - mismatch/dOLRdftot;
     fpnab =fp(nabsorb);
     fpnab = fpnab+dfj;
     fp(nabsorb)= fpnab;
     if abs(mismatch/qtoa)>0.0001
     jf=jf-1;
     else
         jf=nabsorb;
     end
```

```
      if iter>9
         disp('More than 10 iterations for qtoa')
         jf=nabsorb;
      end
    end
  end
end
disp('end of nabsorb loop')

dqdTsK     =   zeros(nods,1);
dqdTsK     =   K*dthetadTsK;
dOLRdTsK   = - dqdTsK(nods)
dqsurfdTsK = dqdTsK(1);
OLRH2O=-q(nods);
qsurfH2O=q(1);
 maxftot=fp(nabsorb);
 maxftot= round(maxftot*10000)/10000;
 fp(nabsorb)=maxftot;
 ftot=maxftot;
  disp('Input for global K&T type heat budget')
  OLRftot=OLR(nabsorb)
  qsurfftot=qsurf(nabsorb)
  qwindowftot=qwindow(nabsorb)

 end        %end of loop for option<3 for water vapor analyses

 if option == 3         %CO2 analyses as compared to the water vapor analysis
                        %often the same variabkle names are used, however
                        %when plotting the resulte "CO2" is added to the
                        %variable names.
 disp('Detailed CO2 analysis for saturation and thermaliztion');
 disp(' for values between ftotCO2=0 and ftotCO2= givenftotCO2')
 disp('                   LIST OF givenftotCO2 values')

disp('givenftotCO2 = 1: saturation value')
disp('givenftotCO2 = 2: twice  the  saturation value ')
disp('givenftotCO2 = 3: three times saturation value')
disp('givenftotCO2 = 4: four times  saturation value')
disp('   ')
iter=1;
while iter>0
givenftot=input('Make a choice for a givenftot finish with 0:')
  if givenftot < 4.1
    if givenftot>0.99
        iter=0;
    end
  end
    if iter==2
    iter=0;
    givenftot=0;
    end
    if iter == 1
    disp('Givenftot should be between 1 and 4')
    disp('Hit any key and try once more')
    pause
    iter=iter+1
    end
end

     %INPUT FOR OPTION 3
```

```
    switch givenftot
     case {1}                %plotting for ftotCO2=1
          nabsorb =20
     case {2}                %plotting for ftotCO2=2
          nabsorb =30
     case {3}                %plotting for ftotCO2=3
          nabsorb =35
     case {4}                %plotting for ftotCO2=4
          nabsorb =40
    end

    nods=100
    fp       = zeros(nabsorb,1);
    qwindow  = zeros(nabsorb,1);    %it includes qabsorb and qwindow
    OLR      = zeros(nabsorb,1);    %Outgoing Longwave Radiation
    qz       = zeros(nabsorb,1);    %make continuous curve of nodal value q(i)
    qtoav    = zeros(nabsorb,1);    %make a vector of qtoa and Prevost
    qPrevostv=zeros(nabsorb,1);     %for plotting purposes
    qzv      = zeros(nabsorb,nods);

   OLRtherm   = zeros(nabsorb,1);
   deltaTsK   = zeros(nabsorb,1);
   forcingOLR = zeros(nabsorb,1);
   dOLRH2OdTsKv =zeros(nabsorb,1);

if givenftot == 1
fmin=0.001;
fmax = 1;
for jf=1:nabsorb
     fp(jf)= fmin+(jf-1)*(fmax-fmin)/(nabsorb-1);
end
fp(nabsorb)=1;
end
if givenftot>1
    nabsorb1=10
    fmin=0.001;
    fmax = 1;
    dftot= (fmax-fmin)/(nabsorb1-1);
    for jf=1:nabsorb1
      fp(jf)= fmin+(jf-1)*(fmax-fmin)/(nabsorb1-1);
    end
    fp(nabsorb1)=1
    fmin=1
    fmax=givenftot;
    for jf=nabsorb1+1:nabsorb
        fp(jf)= fmin +(jf-nabsorb1)*(fmax-fmin)/(nabsorb-nabsorb1);
    end
    fp(nabsorb)=givenftot;
end

    CO2   = zeros(nods,1);
    H2O   = zeros(nods,1);
    fd    = zeros(nods,1);
    znod  = zeros(nods,1);
    q     = zeros(nods,1);
    viewfactor = ones(nods);
    TLR   =zeros(nods,1);
    theta =zeros(nods,1);               %alternative variable for temperature
    qPrevost0 = sigma*TsK^4;            % Prevost flux no water vapor and no CO2 gas
```

```
    qPrevost=fractionCO2*qPrevost0; %Prevost in presence of CO2 gas,
                                    %is plotted as PrevostCO2
    qtherm = 18;                    %fraction of Prevost flux thermalized\
                                    %according to Pangburn 18 out of 46 W/m2


 meshCO2             %  temperature and theta CO2 distribution are calculated
                     %including plotting of distributions of H2O and CO2
                     %densities

dzv=diff(znod);    %from ndal values for z-coordinate make element sizes dz
for i=2:nods-1
  fd(i)=dzv(i-1)*CO2(i);
end
fd(1)=0;                         %normalized distribution of absorption, total
normalized = 1
                                 % at node =1 (surface) and at node =nods
(outerspace)
fd(nods)=0;

totfd=sum(fd);
fd=fd/totfd;
dzv1=dzv(1);
dzvnods=dzv(nods-1);

dOLRH2OdTsK= 3.2342             %from previous runs option 1
heightkmCO2 = round(znodg(4)/1000);

    fmin=0.0001;
    fmax=1;
    if givenftot<1.01
        nabsorb1=nabsorb;
    end
    for jf=1:nabsorb1
         fp(jf)= fmin +(jf-1)*(fmax-fmin)/(nabsorb1-1);
    end

    if givenftot>1
    fmin =1;
    fmax =givenftot;
    for jf =nabsorb1:nabsorb
        fp(jf)=fmin+(jf-nabsorb1)*(fmax-fmin)/(nabsorb-nabsorb1);
    end
    fp(nabsorb)= givenftot;
    end
    jf=0;

    while jf<nabsorb
    jf=jf+1;
    K = zeros(nods);
    f=zeros(nods,1);
    fpjf = fp(jf);
    for i=2:nods-1
       f(i)=fd(i)*fpjf;
  end
 f(1)= 1;          %surface f(1)=1

 f(nods)=1;          % outer space f(nods)=1

  % assemblage of system matrix K
```

```matlab
    makeviewfactor
    makeK
    K = fractionCO2*K;
    q = K*theta;
    qsurfjf= q(1);
    OLRjf = -q(nods);
    qPrevostv(jf)=qPrevost;
    forcingOLR(jf) = qPrevost-OLRjf;
    deltaTsK(jf) =forcingOLR(jf)/dOLRH2OdTsK;
    OLR(jf)  = OLRjf;
    qsurf(jf)= qsurfjf;
    if qtherm > 0                      %to include the thermalisation of CO2
according to Pangburn 18 W/m2
        qthermcor=qtherm;
        if fpjf<1.002
        qthermcor=fp(jf)*qtherm;        % and constant between ftotCO2 = 2 to 4.
                                        %thermalisation distribution linear
between ftorCO2 =1 to 2
        end
        OLRthermjf=OLRjf+qthermcor;     %it could be that the OLRtherm near ftot
= 0 is bigger than qPrevost
        if OLRthermjf>qPrevost
            OLRthermjf=qPrevost;
        end
        qsurfthermjf=qsurfjf+qthermcor;
        if qsurfthermjf>qPrevost
            qsurfthemjf=qPrevost;
        end
        OLRtherm(jf)= OLRthermjf;
        qsurftherm(jf)=qsurfthermjf;
        forcingOLRtherm(jf) = qPrevost-OLRthermjf;
        deltaTsKtherm(jf) =forcingOLRtherm(jf)/dOLRH2OdTsK;
    end

    dOLRH2OdTsKv(jf)=dOLRH2OdTsK;
    qwindowjf=(1-fpjf)*qPrevost;
    if qwindowjf<0
        qwindowjf=0;
    end
    qwindow(jf)=qwindowjf;
 end
 disp('end of nabsorb loop')

 ftotCO2=fp(nabsorb);
 dOLRH2OdTsK                 %defined in wateranalysis
 maxftot = fp(nabsorb);

q=fractionCO2*K*theta;
OLRCO2=-q(nods);
qdisCO2=sum(q)-q(1)-q(nods);
qsurfCO2=q(1);
makeqz
disp('Input for global heat budget')
  OLRftot=OLR(nabsorb);
  qwindowtot = qwindow(nabsorb);
  qsurfftot=qsurf(nabsorb);
 end
switch option
    case {1}
        plotoption1
```

```
        case {2}
            plotoption2
        case {3}
            plotoption3
    end
end
```