Did Newton and Gauss Prove Their Laws?

(Why Are Practical Or Analytical Proofs For Foremost Physics Laws Missing From Mainstream?)

> by M. Nisa Khan, SENIOR MEMBER, IEEE

IEM LED Lighting Technologies Red Bank, NJ 07701, USA Email: nisa.khan@IEM-LED.com Published January 25, 2024

PRINCIPIA SCIENTIFIC



Did Newton and Gauss Prove Their Laws? Why Are Practical or Analytical Proofs For Foremost Physics Laws Missing From Mainstream?

M. Nisa Khan

Red Bank, New Jersey, USA (www.iem-led.com)

Introduction: Did Newton and Gauss Prove Their Laws?

Isaac Newton could not have proven Newton's Law of Gravity that is also known as the Inverse Square Law [1] as debated by the author previously. According to Newton's Principia, Newton offered "philosophy" later inferred by Western scientists as "experimental" philosophy to render Newton's particular propositions as "general laws" by means of supposed "induction".[2]* Proving the so-called Newton's Law of Gravity *analytically* requires the knowledge of advanced calculus in non-Cartesian coordinate systems and analytic geometry [3] and Newton sadly misunderstood even simple calculus in 1-dimensional Cartesian coordinate system.[4] Newton also could not and did not prove the so-called Newton's Law of Gravity *empirically* as this was only later supported by Henry Cavendish, although in an incomplete and hand-waving manner. [5] Cavendish's hand-waving justification came more than a 100 years after Principia was published! Regardless of such an inadequate justification by Cavendish, this law is not universal and does not hold true for arbitrarily shaped objects and distances. The author essentially provided this proof in another publication. So what about other prominent physics laws that claimed to have been discovered by European scientists during and after the Renaissance Period? For example, did Gauss prove Gauss' Laws?

Gauss' Law of electric force field (or energetic field) is symbolically written as,

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0} ,$$

where ∇ (pronounced as Del) is the spatial derivative, \vec{E} is the electric field generated by a differential electromagnetic radiation source containing the electric charge ρ and possessing an inherent permittivity ε_0 .[6] It is the first of the 4 Maxwell's Equations and used in the derivation of the second order differential equation, i.e., the wave equation that governs the behavior of electromagnetic radiation in all space and time. Gauss' Divergence Theorem is symbolically written as,

$$\iint_{S} \vec{F} \cdot \vec{n} \, dS = \iiint_{V} \vec{\nabla} \cdot \vec{F} \, dV \, dS$$

where \vec{F} is the force or energy-flux field diverted out of a bounded source generating the directional force or energy flux-fields that escape through the surface S of the bounded source with volume V as shown in Fig. 1. [7] While both of these laws are used in electromagnetism, the same Gauss' Laws are also analogously used in other areas of physics, such as mechanics including hydraulics and aerodynamics as well as thermodynamics - by appropriately replacing ρ , ε_0 and \vec{E} with their analogs, i.e., suitable equivalent resistance, forces, and other inherent properties relevant for those physical domains or media. These laws are some of the bedrock equations for physics and used extensively in many engineering solutions.



Fig. 1. The illustration of some infinitesimal substance's escaping direction from the bounded volumetric solid V, which is bounded by surface S. This infinitesimal substance that is free to escape from a surface location dS would do so in the direction normal to dS indicated by \mathbf{n} . This substance carries infinitesimal Force \vec{F} or energy flux field \vec{E} at that location.

Have these Gauss's Laws ever been practically proven by Gauss or others during his time or ever since? If so, the author kindly requests the documents, which should have been left by Gauss, or his colleagues or their contemporaries, to be made public. After all, written records became all but common in Europe since the Gutenberg Bible in the 15th century.

What Prompted the Request or Challenge?

Aside from the fact that the proofs for Gauss' Laws do not seem to exist in any existing publications, the author has firsthand extensive knowledge of the sad fact that not a single professional, amateur, or regular person can yet describe what Gauss' Laws say in plain language, e.g., English! In fact, the plain language descriptions of Gauss' Divergence Theorem in this article and that found in the author's book, "Understanding LED Illumination" (CRC Press, 2013), are the first ones to exist in available literature worldwide as of current times! [8] The author obtained this knowledge from her direct interactions with dozens of peer-reviewers called upon by the most notable physics and electrical engineering journal editors [9], her former colleagues from academia and Bell Labs, and from hundreds of responses from social media from around the world in the past 7 years. Thus it begs the question, did Gauss prove Gauss' Laws? The question is similar to, "Did Pythagoras prove the Pythagorean Theorem?" The truth about the latter has been published by Springer in 2009 that Pythagoras in fact did not prove the Pythagorean Theorem.[10] The author also challenges mainstream academia and industries that Euler did not prove Euler's Equation, Fourier did not prove Fourier Transform Relations and Lambert did not prove Lambert's Cosine Law. She humbly requests the academic communities for the written proofs

of these by the original authors or discoverers because one would expect the prestigious European universities to have recorded such.

Western-Developed Mathematics and Physics Rely on Faith

After some extensive original work in the areas of light and EM radiation and years of literature search from other notable mathematicians and physicists, the author notes that mathematics became a formal discipline decoupled from physics or other scientific fields in modern times; likewise physics became decoupled from mathematical laws that are not justified from empirical demonstrations. Academics from the recent past and current mainstream appear to have adopted mathematics that has been manipulated by the Western schools of thought, which treated mathematics as a set of postulates and hypotheses supported by presumptive axioms that are often taken as theorems written by symbolic, denoted formulas – but not in plain spoken or written language used for everyday communication among humans in society. Such axioms have only been depicted with mathematical symbols without describing them in plain spoken language and without any empirical, i.e., practical proof form the very start of Western physics and mathematics. These axioms formed the beginnings of modern science, mostly credited to the West, are bothersome to say the least. Educating all students in this manner then surely appears very dishonest. Further, such education practice has proved to be difficult for most people to grasp physics and mathematics.

The Western schools offered proofs for only some of their mathematical theorems albeit with the use of another axiom or a set of axioms that were never proven to be true empirically by any Western scholars or anyone from the West since Western colonization of the world took off. Axioms are taken to be true on an assumptive basis and not on the basis of any analytic reasons or from empirical demonstrations. If any empirical or purely logical or comprehensive analytical proofs existed for their theorems many of which are well-known and often used in physics, they would have kept such records in the notable European universities and towns where many of those mathematicians worked and lived only a few centuries ago.

Pythagorean Theorem was never proven by Pythagoras [10] and all of its currently available proofs, some of which existed long before Pythagoras, make use of geometry representing physical attributes and physical demonstrations. In other words, the so-called Pythagorean Theorem is true because it can be empirically demonstrated and not because Pythagoras said so or used it or found it among ancient treasures. Fourier could not have empirically proved FT relations during his time and if he did, either practical demonstrations or purely analytical proofs with meaningful descriptions without the usage of other proven mathematical theorems would have been recorded and valid proofs would have been common knowledge at least within the scientific and mathematical communities. The fundamental governing physical laws lacking empirical proofs have established the current norm in 'Western colonized physics' and this is very disturbing. Among the foremost laws in mainstream physics, it is all but arguable that Newton did not provide any proof for his Law of Gravity [5] and Johann Lambert did not provide any proof for the Law of Cosine for flat radiators as discussed later in this paper. Such governing physical laws, named after notable physicists, have been taken to be true by faith in the 'colonized physics' dogma only because partial empirical proofs under special circumstances

only, if any, were established many years after the laws have been supposedly established by those notable scientists and mathematicians.

Ganita or Bharatiya Mathematics Before Macaulay is Not Based on Faith

Unlike today's mainstream academic mathematics, the mathematics discipline as referred to as 'ganita' in ancient Bharatvarsha that now stands as separate countries in the Indian Subcontinent, used empirical proofs for many mathematical formulas that are known and utilized today. Ganita included number systems and counting methods now known as Arabic Numerals; it included algebra, geometry, trigonometry, calculus, and much more. Ganita supported physics, and physical, i.e., practical proofs were used as demonstrations to establish mathematical relations or formulas. [11] As of now, evidence for a rich set of examples of Bharatiya ganita and biggyan (science) can be found to have existed long before the Muslim World and European Christian society repackaged much of the same work albeit erroneously. [12, 13] The author cautions that ganita today in India refers to mathematics that is synonymous with Western mathematics and therefore one must differentiate ancient Bharatiya ganita from 'ganita' today. However, there is a widespread misunderstanding today that ancient Bharatiya ganita is synonymous with "Vedic ganita" and this reference to Vedic ganita or Vedic mathematics is often associated with certain arithmetic tricks or shortcuts. [14] The author refers to ancient Bharatiya Ganita as that which established mathematical equations based on natural laws by means of practical proofs and that which developed into a rich set of quantification, description and qualification dogma or fields <u>and</u> methods to assess natural substances and their behaviors. These fields and methods fall into the subsets of the entire ganita discipline that included algebra, geometry, trigonometry, calculus and others.

Why Should We Care Now?

So why does it matter if much of today's mathematics and physics are newer versions of the more ancient mathematics and physics? It matters because Europe systematically syphoned Indian mathematics, physics and general scientific knowledge, starting mostly during the 15th century when the European Christian Church began their pedagogic establishment in India to spread Christianity. [12] [13] However, during this process of scientific knowledge transfer, some very crucial concepts in mathematics and physics were misunderstood at the very fundamental level! One such concept is how basic mathematical equations or relations represent physical reality under certain physical circumstances – usually in specified domains of physical space and time continuum. The West failed to understand that ganita and physics are one and the same because ganita uses natural physical phenomena for establishing mathematical formulas thereby every valid mathematical formula is a physical representation of a certain physical system.

The basic Pythagorean Theorem for example is only valid over flat planes and not valid in any curved spatial domain or curved surfaces. It is therefore crucial to prove the Pythagorean Theorem using practical methods, which will then invariably demonstrate that the formula only is valid over flat planes. While this is now an easy example that demonstrates the importance of how mathematical theorems ought to be established or as some say, discovered, this concept was not embraced as a

necessity in Europe when many European mathematicians and physicists adopted what are now known as modern or imperial or colonized math and science. Despite the West claiming math and science are not religious, their math and science actually became faith-based for which major equations were embraced only as axioms that were dubbed as some great scientists' or mathematicians' laws, without proper practical proofs that validated them for certain physical conditions.

Only Ganita's Calculus Adopted by Aryabhata Can Offer a Rescue

Such faith-based association for Western science and mathematics is especially true for calculus! Neither Newton nor Leibniz understood the fundamental concepts behind calculus and did not demonstrate why and how calculus is a necessary and sufficient method of calculation for *any* general physical substance such as energy and matter in any user-specified space and time and space-time! The author has challenged the physics community that Newton could not have proved the so-called Newton's Law of Gravity because this requires proficiency in calculus in 3D spherical coordinates that Newton knew nothing about according to kept records.[3]

In fact the crucial concept of calculus for a general problem that requires the usage of the most appropriate coordinate system and the usage of an appropriate infinitesimal have gone amiss in modern science and mathematics. Could this lack of aptitude be directly related to deliberately holding Newton at a 'sainthood' status by Western science and mathematics communities? How else can one explain the current modern science and mathematics philosophies, which have been primarily shaped and dominated by the European scientists and mathematicians since the Renaissance Period, make use of a 'point mass' concept with no volume for physical entities like particles and waves? Why most professionals and regulars alike are unable to solve general quantification problems using finite sizes and shapes in physics analytically? Further, why numerical simulations of physics and engineering problems only use Cartesian coordinates even when curves at very small or an arbitrarily small scale are present in a practical problem? It is also bothersome to note that hardly any scientist or mathematician recognizes that when anyone performs a numerical calculation or simulation either by hand or by means of computer algorithms, one is invariably solving a finite and bounded problem. This blunder adds to the already troublesome theory of quantum mechanics when every graphical representation of quantum states and discreteness are routinely presented by means of numerical simulation. The author intends to elaborate on the contradictions and blunder of quantum mechanics in the future.

Ganita is Essential for Valid Measurements

What is even a bigger concern today is that the measurement methods that became the norm in physics, general science and engineering only work within limited scope because every physical measurement must apply appropriate calculus concept and techniques and not some generic calculation concept and techniques. For example, for an arbitrary physical substance measurement, one cannot apply the same calculation techniques that make use of only calculus in either Cartesian Coordinates or avoid using any calculus altogether by only using the 'point' concept that is taken to be dimensionless spatially. Such practice is erroneous for practical physics and poses a great danger to our lives and environment because grossly inaccurate calculations and assessments can lead to calamities of all sorts.

As long as scientists apply their measurement techniques within limited scenarios for which their analysis and measurements are by and large appropriate and precise enough, the results are usually acceptable. However, if the same measurement techniques and algorithms are used for physical conditions where different calculus analyses using comprehensive 3D-analytic geometry are necessary, the obtained laboratory or field results will not at all be valid or sufficiently precise. This scenario is similar to the inapplicability of the Pythagorean Theorem in some 3D space-time curvature. The author offers an online tutorial on this subject at https://dr-nisa-s-site.thinkific.com.

Primary examples of such grave mistakes in measurement and calculation can be widely noted in illumination from LED car headlights and many outdoor and stadium lights where the horrendous glare from these lights or luminaires are witnessed by many people around the world.[15] Although microwave and RF radiation is not visible, similar problems also exist for today's smart phone antennas and base station antennas that generate excessive radiation from 4G and 5G signals. These radiation devices are making many people sick and debilitated.[16] The academic and industry authorities for these devices and technologies are IEEE, IES, CIE, ICNIRP, NHTSA, UL, NIST and others. They are all making a grave mistake when they either confine themselves to only Cartesian coordinate calculations or treat flat radiators such as LEDs, lasers, and cell phone antennas as dimension-less 'point' radiation sources without defining their flat structures. Surely they need to recognize that a point-source radiator is only spherical like the sun, which spreads radiation uniformly in all directions in 3D space and thus has *zero* directivity as shown in Fig.2.



Fig. 2. A single LED source is always flat as shown in A1; in A2, the radiation pattern off the surface of a flat LED is shown; although the 3D radiation pattern in A2 is spherical in shape, the light density or luminous intensity within this sphere is very nonuniform as shown using false multi-colors and intensity in every direction away from the center axis drops off substantially; consequently a directed beam is

generated off the surface of the LED as shown in A3 where density profile in one direction is shown in gold curves that resemble Gaussian functions. In sharp contrast, the sun is spherical as drawn in B1; sun rays emanating off its spherical surface is shown in B2 and all the space illuminated all around the sun is uniform; consequently light seen far away from the sun has nearly perfectly uniform illumination in 3D space as seen in B3. The drawing here is not to scale.



Fig. 3. LEDs, uncollimated laser and collimated lasers all have finite directivity that can be calculated and measured. Typically a collimated laser has much higher directivity than an uncollimated laser and a LED's directivity is less than that of a laser as shown here. In contrast, the radiation from the sun has zero directivity, which is strictly an attribute of a spherical radiation source. The drawing here uses false colors and is not to scale.

In contrast, every flat radiator shows a directive behavior as shown in Fig. 2 and Fig. 3, where their directivity is finite and can be quantified, and this is a violation of a 'point' source radiator at the fundamental level. Such violation is overlooked by mainstream academia and industries today because modern physics and mathematics, shaped by the Western World, has been under false pretension that Newton proved his Inverse Square Law of Gravity in some valid manner with or without the augmentation from Cavendish! It is a false pretension because neither one used calculus that is necessary to measure or analyze this so-called Universal Law of Gravity.

Is illumination from a flat radiator supposedly described by Lambert by means of his Cosine Law? It is not clear to many that it is the case. Even if the best mainstream physicists and engineers from the optics and lighting industries state that the light distributions from LEDs are Lambertian, do they know what that means and how such a radiation pattern is different from that created by the sun? The author repeatedly experienced from these industry leaders that they do not know what a Lambertian radiation pattern in 3D space means and how to visualize it in 3D space. Thus it begs the questions: did Lambert ever measure a flat radiator's directivity? Did he ever prove his so-called Lambert's Cosine Law? What is Lambert's Law for a finite, flat radiator? Did he know what the Lambertian radiation pattern looked like in 3D space? Did he know that such a radiator invariably produces a directive beam? If so, where are these proofs or written descriptions that a Lambertian radiator has a finite directivity? The author respectfully requests the Swiss and German esteemed universities and academic communities to provide that Lambert either analytically or empirically proved his law since Lambert was a Swiss-German scientist in the 1800's and recording work of great importance during that time was all but common in his geographical region.



Fig. 4. A schematic similar to that which describes Lambert's Cosine Law in existing literature is shown.[17] According to Lambert's Cosine Law, a differential element dA produces luminous intensity, *I*, as seen by the viewer from different angles, ϕ . False colors are used in this diagram.

The schematic shown in Fig. 4, depicted only in 2D for a non-finite radiator, is <u>not</u> sufficient for obtaining its 3D generalized version, empirically or analytically and thus begs the question: "Did Lambert prove his law analytically or empirically?"

Ganita's Calculus Imperative for Proving Lambert's Cosine Law

According to the author's knowledge based on numerous erroneous peer-reviews she received over the course of five or more years [8], the proof of Lambert's Law by Lambert does not exist. It could be the reason why so many mainstream physicists, mathematicians, electrical engineers and optical scientists really do not know what the law is saying in plain language. It appears that the author is the first to prove Lambert's Law for both an infinitesimally flat radiator and for a finite, flat radiator.[18]

Lambert's Cosine Law could not be proven without advanced calculus and analytic geometry that utilizes Cartesian as well as spherical, polar, and cylindrical coordinates to solve a finite problem encountered in a physical system that can be also verified empirically.[18] Lambert simply did not and

could <u>not</u> have done either. If anyone has proof of the contrary, they should present to the public how Lambert proved Lambert's Law either analytically or empirically. Further, the generic Lambert's Cosine Law that exists in the mainstream literature is incomplete in that it does <u>not</u> show that the magnitude of the peak optical power at the center of the flat radiator is related to the radiator's inherent parameters such as the size of the flat radiator and its inherent radiance for the infinitesimal part of the flat radiator. In contrast, the author has demonstrated that a suitable infinitesimal representing infinitesimal radiative power from a flat radiative source must be integrated over the finite size of the radiator to prove Lambert's Cosine Law for a finite, flat radiation source as shown Fig.5.



Fig. 5. The radiative intensity of a flat radiator must be obtained by integrating a differential radiative power on the differential surface of a flat emitter over the entire surface of the radiator. [18]

In Fig.6, the calculated 3D radiation pattern from a flat radiator based on author's derived formula is shown and such a pattern is precisely nothing but a 3D Lambertian.[19] There is now empirical proof for the generalized Lambert's Cosine Law for a finite size flat radiator with its inherent radiation properties as seen in the measured data in Fig.7 [18], showing graphical agreement with author's calculation of 3D Lambertian radiation pattern and that obtained from an industrial empirical measurement.



Fig. 6. The Lambertian radiation pattern from a flat radiator calculated using the author's derived equation. The graph uses false colors. [19]



Fig. 7. The Lambertian radiation pattern from a flat radiator measured using a goniophotometer. [18]

Pathological Science and Mathematics Causing Pathologic Harm to Species

The optical, lighting and the RF academic and industrial communities have been simulating the near-field of surface radiation intensity of flat radiators using Cartesian coordinate systems exclusively. While this is a valid approach to determine the radiative power distribution on the surface of a flat antenna or an LED or a laser, no one from the mainstream RF wireless, waveguide, or antenna community can simulate what the radiative power distribution is in 3D near-field just following the antenna, or an LED, or a laser. Why not? Because these scientific, technical and mathematical communities do <u>not</u> know how to solve the analytic problem for 3D near-field radiation emitted off a flat antenna or a generic radiator! Still, miraculously, the entire RF simulation community is aware of its far-field 3D radiation pattern! The author challenges the RF community to come clean and explain why they have decided that the far-field simulated radiation pattern of a flat antenna is a Lambertian as shown in a typical simulation.[19]

The author would like to know whether the RF community calls such a radiation pattern a Lambertian or a general cardiod. The author challenges the RF community to reveal how it has decided on such a 3D radiation pattern at far-field because such a radiation pattern can only be obtained by means of near-field measurements or propagating a near-field simulation to far distance. Have they proved such is the radiation pattern analytically or empirically or using yet another axiom from somewhere? Where is the proof that a far-field radiation pattern in 3D from flat antenna is a Lambertian or a weighted Lambertian or a cardiod? This challenge is presented to the RF simulation and measurement communities because the author has analytically proven that only an equation describing a cardiod can produce such a 3D radiation pattern. The RF communities, existing for over a century, pulling such scientific and mathematical information out of nowhere yet once again is gravely concerning. The author has numerous other challenges for these scientific communities regarding radiation energy and power from flat and other radiation sources and these will be discussed in future publications.

We Can Create Our Better Future

The author's main interest in getting the message of this article to the public as a set of challenges to the esteemed, mainstream physics and electrical engineering community - is to meet her moral obligation to the millions and billions of people suffering from outdoor LED lighting including automotive LED lighting and consumer wireless smart phones. She has received dire messages from many grown-up men who lost their good jobs because of facing LED lights in public places. She is often contacted by sufferers of smart phone radiation. In order to help such helpless communities of real people around the world, the author respectfully presents her challenges laid out in this article. If the mainstream physics and electrical engineering academic and industry circles are not able to meet her challenges in a public forum or several such forums in any decent timeframe, the author humbly asks to immediately ban all LED outdoor lights, especially LED car headlights; also to stop using consumer wireless.

References:

1. https://en.wikipedia.org/wiki/Newton%27s_law_of_universal_gravitation

(Accessed on January 22, 2024.)

2. (* The author recognizes that this Wikipedia reference may not exist in the future. Therefore, the original reference used to establish Newton's Law of Gravity is provided:

Isaac Newton: "In [experimental] philosophy particular propositions are inferred from the phenomena and afterwards rendered general by induction": "Principia", Book 3, General Scholium, at p.392 in Volume 2 of Andrew Motte's English translation published 1729.

* The author asserts that induction proof cannot be applied to prove a physical phenomenon experientially if the physical phenomenon is a law of nature that relates on physical parameter to another parameter in a quadratic fashion or via higher order. Induction proof is only valid for linear relationships between two physical parameters.

3. <u>https://principia-scientific.com/where-are-the-proofs-for-newtons-law-of-gravitation-and-coulombs-law/</u>

(Accessed on January 22, 2024.)

4. <u>https://ckraju.net/papers/Calculus-story-abstract.html</u>

(Accessed pm January 22, 2024.)

5. "<u>The Michell–Cavendish Experiment</u>, by Laurent Hodges" is used as a reference in Ref. 1. However, this reference is no longer available. Such an important experiment's record should nevertheless be available in the University of Cambridge.

6. J. D. Jackson, "Classical Electrodynamics", John Wiley & Sons, New York, 1975, Chapter 1, Section 1.3, "Gauss' Law", pp. 30-33.

7. E.W. Swokoski, "Calculus with Analytic Geometry", 2nd Edition, Prindle, Weber, and Schmidt, Boston, Massachusetts, 1981, pg. 929, Chapter 18.

8. M. Nisa Khan, "Understanding LED Illumination", CRC Press, 2013, Chapter 6.

9. <u>https://www.researchgate.net/profile/M-Khan-36</u> (Rebuttal Articles on Optica, APS, IEEE, and other peer-reviewed journal editors' and reviewers' comments.)

10. https://doi.org/10.1057/jt.2009.16

(Accessed on January 22, 2024.)

11. <u>https://ckraju.net/papers/IIAS-Seminar-abstract.pdf</u>

(Accessed on January 22, 2024.)

12. C. K. Raju, "Refutation of the Aryan Race Conjecture", Kant Academic Publishers, Delhi, 2022.

(ISBN: 978-81-909161-0-3)

13. CKR, "Eternity and Infinity: the Western misunderstanding of Indian mathematics and its consequences for science today." American Philosophical Association Newsletter on Asian and Asian American Philosophers and Philosophies 14(2) (2015) pp. 27–33. <u>http://ckraju.net/papers/Eternity-and-infinity-Pages-from-APA.pdf</u>.

14. <u>https://www.thehindu.com/opinion/op-ed/nothing-vedic-in-vedic-maths/article6373689.ece</u>

(Accessed on January 22, 2024.)

15. https://lightaware.org/blog/why-leds-are-uniquely-unsuitable-for-vehicle-headlights

(Accessed on January 23, 2024.)

16. <u>https://news.berkeley.edu/2021/07/01/health-risks-of-cell-phone-radiation</u>

(Accessed on January 23, 2024.)

17. <u>https://en.wikipedia.org/wiki/Lambert%27s_cosine_law</u>

(Accessed on January 24, 2024.)

18. M. Nisa Khan, "Derivation and Experimental Verification of the Near-field 2D and 3D Optical Intensities from a Finite-size Light Emitting Diode (LED), IEEE Photonics Journal, 11(6), December 2019.

19. <u>https://principia-scientific.com/wp-content/uploads/2022/03/khan-paper.pdf</u> (Accessed on January 24, 2024.)