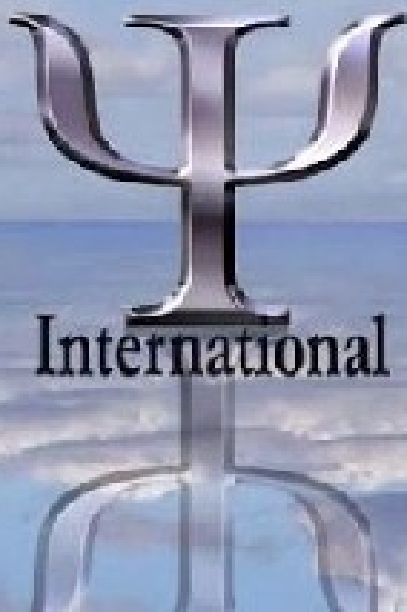


Diurnal variations of heat evacuation from rotating planet

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Diurnal variations of heat evacuation from rotating planet

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updated Sept 2014, numerical examples in appendix 1, MATLAB listing appendix 2

Introduction

In earlier papers [1, 2, 3, 4] the author has discussed one-way versus the two-way formulation for heat evacuation by radiation from the planet, in steady-state conditions.

In [1] is presented a model of the semi-transparent atmosphere consisting of a stack of gauzes, representing the IR-active trace gases (molecules with three or more atoms). Results have been validated by comparing them with published results of K&T type of diagrams based on two-way formulation. The one-way formulation does not show the huge absorption in the atmosphere nor the non-physical back-radiation therein.

In [2] a FEM (finite element method) implementation is presented for the same model with the same results as [1] concerning OLR and sensitivity of CO₂: doubling the concentration from the present 0.04% to 0.08% would give an increase of 0.03°C.

In [3], again by means of FEM, the heat transport by conduction through two slabs with a finite thickness, separated by a vacuum with a radiation heat transfer, shows once more that back-radiation of heat from cold to warm cannot exist.

In [4] a comparison is made with the Schwarzschild approach from the beginning of 1900 as advocated by IPCC, from which follows that the Schwarzschild approach is in fact also a one-way formulation - but wrongly interpreted by IPCC authors!

The LW flux upward flux is split up artificially in an up-going flux U and a down-welling flux D, the latter taken positive in the negative z-direction. The two LW components follow indeed the same path but in opposite direction!

In the beginning of the 1900's computers were not available and by splitting-up the radiation in up-ward and down-ward components, and introducing a co-ordinate transformation with the so-called optical thickness concept, analytical solutions were possible, although in the form of integrals. Quadrature techniques were available at that time to evaluate numerically those integrals, with no need for computers.

In [4] it is shown that IPCC authors make a wrong interpretation by trying to give a physical interpretation of the components U and D, by evaluating an artificial absorption

for U and another one for D, instead of the absorption for the real flux $R = U - D$. As argued by Claes Johnson [5] heat-flow by radiation is from warm to cold! Electromagnetic radiation in two directions transfers the information between the warm surface and the cold one, on the basis of which heat transport from “warm to cold” is established. See also the history of back-radiation by Matthias Kleespies [6] The main conclusion of the earlier papers [1, 2, 4] is that the heat evacuation from the surface in the SS condition of the global and annual mean heat budget is not that much by radiation but rather by convection of sensible and latent heat as is shown in figure 1.

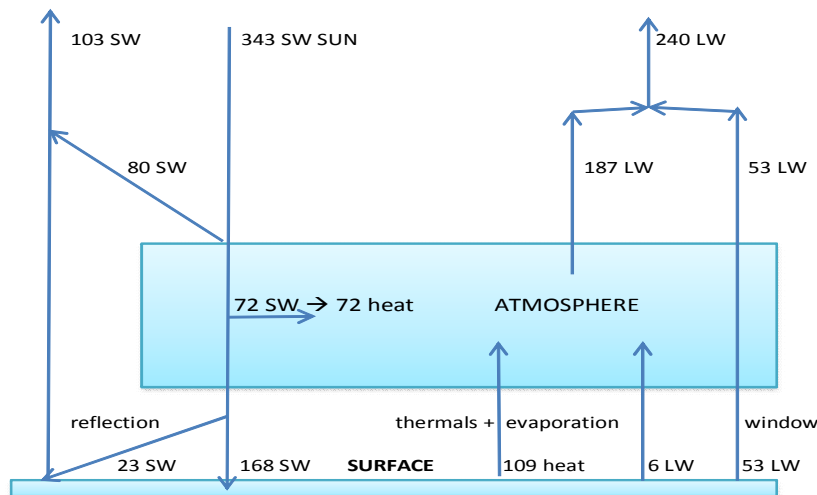
Objections are forwarded to the mono-chromatic steady-state model.

Figure 1 gives the global and annual mean heat budget of the planet as obtained by the stack-model [1, 2, 4] and validated in [1] by comparison with IPCC publications. From the 343 W/m^2 which is the average SW flux from the sun at the top of the atmosphere of the planet, 103 are reflected as SW flux by the atmosphere (80) and by the surface (23). From the remaining 240 SW, the atmosphere absorbs 72 W/m^2 and 168 are absorbed by the surface.

Only 59 W/m^2 leave the surface as LW radiation, 53 through the atmospheric window straight away to outer space and a mere 6 W/m^2 LW radiation is absorbed by the atmosphere. The remaining 109 W/m^2 are leaving the surface by convection of sensible and latent heat.

Figure 1

Global and annual mean heat budget in W/m^2



- (1) Opinions are forwarded that a mono-chromatic model would not be representative. But nobody gives the real dependence of absorption of the traces of the IR-active gases on the wavelength. And how can one have confidence in experimental people who claim they have measured back-radiation of heat, from cold to warm!
- (2) Opinions are forwarded that the averaging procedures would be of little use for a rotating planet. Again without any argumentation.

Ad (1)

The heat evacuation from the surface of the planet in SS to the top of the atmosphere (TOA), is not that much governed by radiation but rather by convection of sensitive and latent heat.

The LW radiation from the surface is a mere 6 W/m^2 , apart from the radiation through the atmospheric window of 53 W/m^2 . The global and annual mean budget as presented in figure 1 is nearly equal to the one presented by IPCC authors, if one subtracts from their claimed atmospheric absorption the non-physical back-radiation, see [1].

The LW surface radiation of 6 W/m^2 , absorbed by the atmosphere and re-emitted again, is the only flux with a spectral dependence of absorption by traces of IR- active gases.

The 6 W/m^2 can be neglected in comparison with the 109 W/m^2 heat from the surface by the mechanism of convection plus the 53 through the atmospheric window.

Ad(2)

The scope of the present paper is to show the correctness of the FEM stack model based on the identification of pairs of emitters and absorbers by means of finite elements with two nodes. A heat flow by radiation from the warmer node to the colder one by Stefan-Boltzmann turns out to be an excellent description of the SS (steady-state) evacuation of heat from the planet.

As will be shown in this paper the model is sound and able to deal with the diurnal variation of the incoming heat from the sun, due to the rotation of the planet: simply by identifying fast and slow mechanisms.

Another confirmation that the stack model is correct in the conclusions of it, that the absorption of 0.04% of CO_2 can be neglected with respect to the absorption by water vapour.

Besides, a higher CO_2 concentration is needed, to promote an increased growing of plants to feed the increasing world population.

The radiation model

The stack model as developed in [1, 2, 4] defines for SS conditions global and annual mean evacuation of heat by the matrix equation:

$$\mathbf{q} = \mathbf{K} * \boldsymbol{\theta} \quad (1)$$

Here \mathbf{K} is a radiation matrix, in a model with 40 nodes of dimension 40×40 .

The components of \mathbf{K} depend on the absorption coefficients \mathbf{f}_i with a total $\mathbf{ftot} = \text{sum}(\mathbf{f}_i)$.

The quantity \mathbf{ftot} is also called the optical thickness at the surface, representing the total amount of IR-active gases in a column of air of the atmosphere.

The variables in (1) are $\boldsymbol{\theta} = \sigma \mathbf{T}^4$, where \mathbf{T} is the vector of the measured temperature distribution in the atmosphere, corresponding very well to a distribution defined by the environmental lapse rate $\mathbf{ELR} = - 6.5 \text{ K/km}$ and the surface temperature \mathbf{T}_s :

$$\mathbf{T}(z) = \mathbf{T}_s + \mathbf{ELR} * z \quad (2)$$

Figure 2 shows the measured temperature distribution for the standard atmosphere, as well as for tropical and polar regions. We see that the 3 slopes of the temperature dependence are nearly equal: the environmental lapse rate $ELR = dT/dz = -6.5 \text{ K/km}$. In the polar region, an inversion is observed.

Such inversions, but on a lesser scale, occur also near the surface of the planet in the early morning around sun rise, as has been analyzed by the transient model of this paper.

Figure 2 from the Public Domain Aeronautical Software [7]

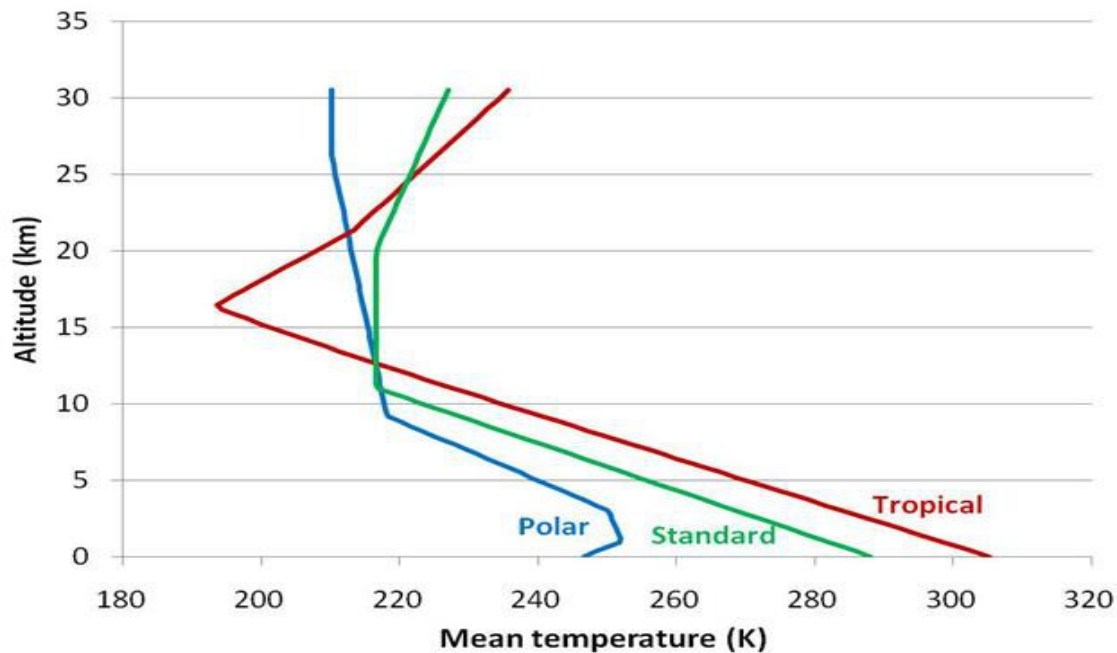


Figure 3 gives a graphical display of the relation $\mathbf{q} = \mathbf{K} \cdot \boldsymbol{\theta}$.

The components of the vector \mathbf{q} in the relation (1) represent the flux into the model, necessary to obtain the measured temperature distribution, with the first component into the atmosphere $\mathbf{q}(1) = \mathbf{q}_{surf} = 59 \text{ W/m}^2$ and the last component out of the atmosphere $\mathbf{q}(nods) = -OLR = -240 \text{ W/m}^2$.

The remaining components of \mathbf{q} are the necessary deposits of heat in the atmosphere by “mechanisms other LW radiation” in order to obtain the measured value for $\boldsymbol{\theta} = \sigma \mathbf{T}^4$: in total 181 W/m^2 of which 72 as SW absorption by aerosols in the atmosphere and 109 as convection of sensible and latent heat from the surface.

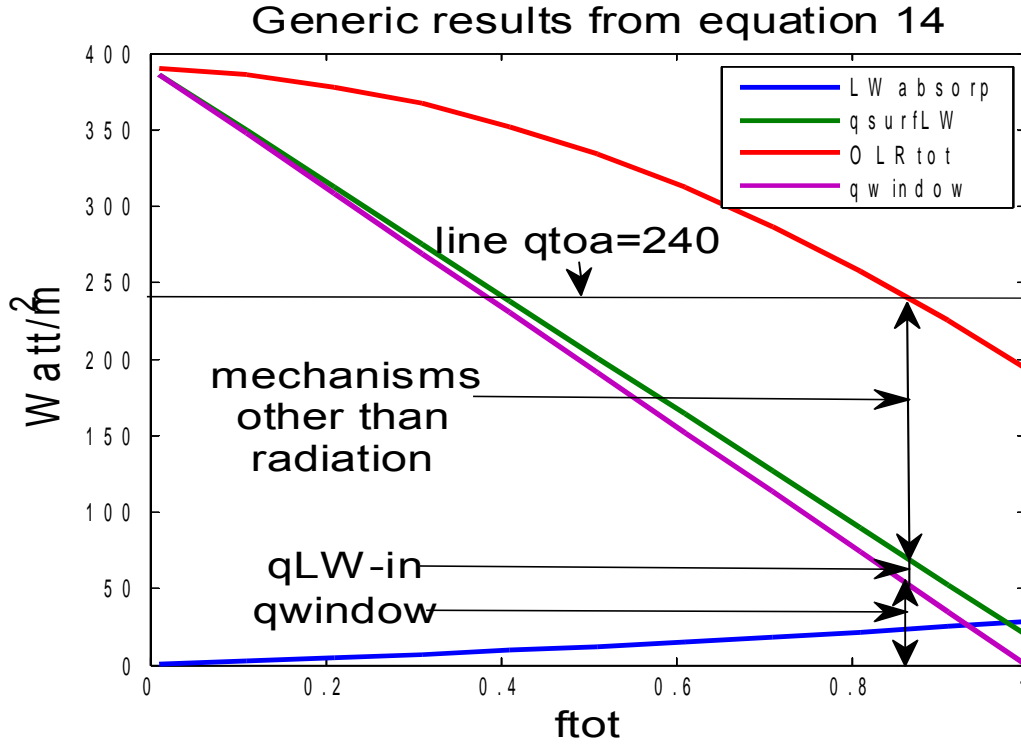
The bigger part of the LW surface flux is the flux through the atmospheric window. For the global and annual mean $\mathbf{q}_{window} = 53 \text{ W/m}^2$. It is important to notice here that the flux through the atmospheric window does not depend on the temperature of the atmosphere.

Indeed it is a Prevost type of flux defined only by the surface temperature \mathbf{T}_s , the surface emission coefficient $\boldsymbol{\varepsilon}$ and the value $(1 - \mathbf{ftot})$, where $\mathbf{ftot} = 0.86$ is the optical thickness or

the sum of the absorbing IR-active molecules in a column of air mainly water vapour:

$$q_{\text{window}} = (1-f_{\text{tot}})\epsilon\sigma T_s^4 \quad (3)$$

Figure 3 Results of equation $q = K*\theta$



$$LW_{\text{absorb}} = \text{sum}(\text{INTO} * \theta_{\text{LR}})$$

$$q_{\text{window}} = (1-f_{\text{tot}}) * \epsilon_1 * \sigma * T_s^4 = (1-f_{\text{tot}}) * \epsilon_1 * \theta(1) \quad (4)$$

$$q_{\text{surfLW}} = q(1)$$

$$OLR_{\text{tot}} = -q(\text{nods})$$

Note the low value of the LW radiation into the atmosphere, $q_{\text{LW-in}}$.

It is going to be absorbed by IR-active gases, re-emitted and eventually absorbed and re-emitted again, since $LW_{\text{absorb}} \approx 1.5 * q_{\text{LW-in}}$.

Figure 3 gives all the values as used in the global and annual heat budget of figure 1. The various fluxes are plotted as function of f_{tot} .

Drawing the line $q_{\text{toa}} = 240$, gives an intersection at the global and annual mean value $f_{\text{tot}} = 0.86$. Figure 3 has been established for a global and annual mean surface temperature $T_s = 15 \text{ C} = 288 \text{ K}$.

Mechanisms other than LW radiation consist of atmospheric absorption of incoming SW radiation and of convection of sensible and latent heat, vertically by thermals and horizontally by wind.

Global heat balances for alternative temperature distributions

The stack model as described in the previous section, with the global and annual mean heat budget, is a SS model for a temperature distribution according to the measured environmental lapse rate and a measured mean surface temperature.

In figure 4 are shown 3 different atmospheric transient temperature distributions as possible variations of the original SS distribution with lapse rate $ELR = - 6.5 \text{ K/km}$ and surface temperature $Ts_0 = 288 \text{ K}$.

Case 1:

The atmospheric temperature does not change with the surface variation ΔTs :

$$TLR(1) = TLR0(1) + \Delta Ts \quad (6)$$

$$TLR(i) = TLR0(i) \quad \text{for } i > 2$$

Case 2:

The atmospheric temperature is determined by the lapse rate starting from $Ts_0 = 288$ with a variation of the near surface temperature with an exponential decay:

$$TLR(i) = TLR0(i) + \Delta Ts * \exp(-z(i)/zrelax) \quad (7)$$

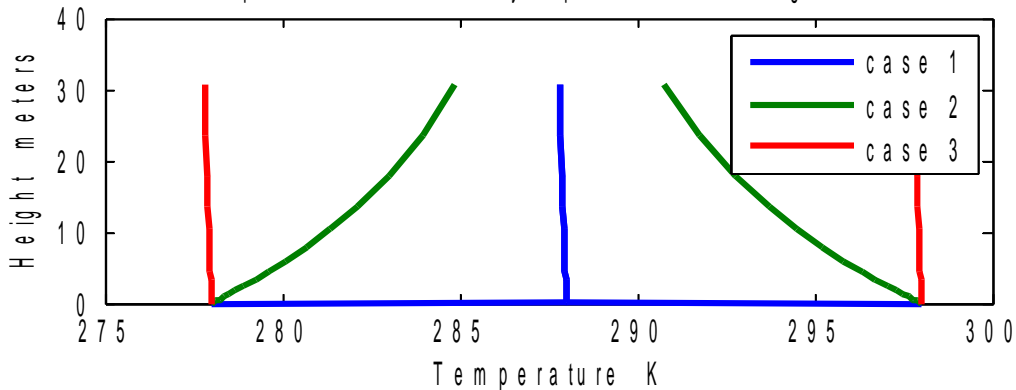
Case 3:

A translation of the temperature distribution with the surface temperature similar to the curves of figure 2 for the standard atmosphere, the tropical one and the polar one:

$$TLR(i) = Ts_0 + \Delta Ts + ELR * z(i) \quad (8)$$

Figure 4

fig 2.9 Steady state solutions for various atmospheric and surface temperature
 case 1: atmospheric lapse rate, atmospheric T does not change only surface T
 case 2: $\exp(-z/relax)$ decay towards lapse rate mixlair= 25
 case 3: atmospheric T defined by lapse rate starting from new surface T



The temperature distribution of Case 1 is the extreme for the initial conditions of the transients in the atmosphere. It is fast in the sense that the first node of the atmosphere follows the temperature of the surface defined by a diffusion process in the sub-surface

and incoming sun power.

The temperature distribution of case 3 is the other extreme. It is supposed that the complete column of air has changed temperature according to the surface temperature T_s and the environmental lapse rate ELR. This process is slow.

In figure 5 the results are given for SS results for the extreme case 3. The fluxes are not valid for diurnal variations of the surface temperatures which do not involve temperature changes of the complete column of air above the atmosphere.

Figure 5

fig 2.7 Steady state solutions for various atmospheric and surface temperature
 Atmospheric temperature according to lapse rate $LR = -0.0065$
 Starting from surface temperature (x-axis)

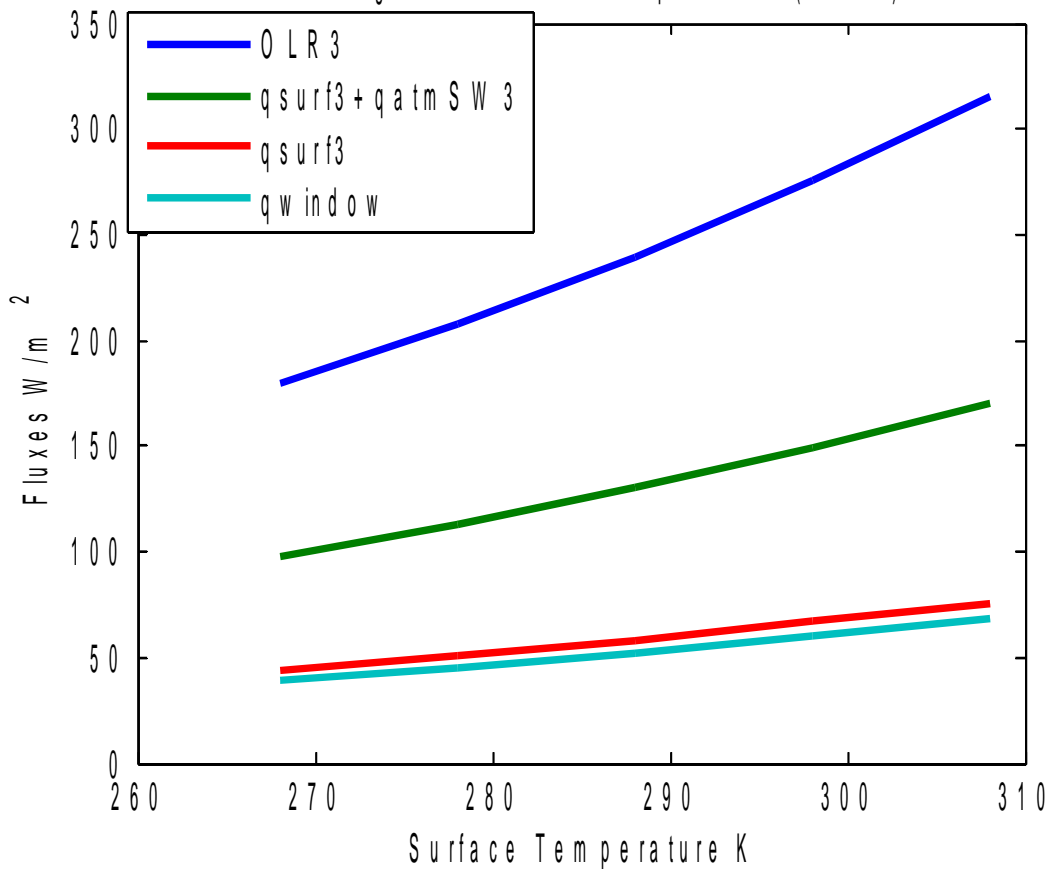


Figure 5 shows that global and seasonal mean heat budgets for different surface temperatures follow from values which change gradually. We see that the LW radiation from the surface remains small of the order of 6 W/m² as for the global and annual mean heat budget with a surface temperature of 288 K. (figure 1).

The temperature variation of the complete column of air is a slow process due to the heat capacity $\rho_{air}H$ of the column:

$$\rho_{\text{air}} H = \int_0^H \rho c dz = \rho c/g \approx 10^7 \text{ J/m}^2/\text{K} \quad (9)$$

$$p = \int_0^H \rho g dz \approx 10^5 \text{ N/m}^2 \quad \text{pressure at the surface}$$

For an average sun power of $q_{\text{sun}} = 240 \text{ W/m}^2$, in daytime from 06h00 to 18h00 the average sun power is 2×240 which gives again the 240 for heating since 240 W/m^2 is the average heat loss to outer space. The increase of the temperature of the column, if the heating would be homogeneous, during the 12 hours of the day, would be :

$$\Delta T = 12 \cdot 3600 \cdot q_{\text{sun}} / \rho_{\text{air}} H \approx 1 \text{ }^\circ\text{K} \quad (10)$$

For a land surface this daily variation is far too low. Indeed not the complete column participates in the diurnal variation.

For a sea surface the determination of the daily variation is of the order of $1 \text{ }^\circ\text{K}$, but that low variation of a sea surface is due to the amount of sea water which takes part in the daily diurnal temperature variation mechanism. In this respect it is useful to compare the heat capacity of the column of air with the heat capacity of 1 cubic meter of water or a column of 1 meter of water:

$$\rho_{\text{water}} = 4.2 \cdot 10^6 \text{ J/K/m}^3 \quad (11)$$

We see from (9) and (11) that a layer of 2.2 meter of water has the same heat capacity as the total column of air! However, since for the water dimensions are much smaller, as well as due to the mixing mechanisms of surface waves, the efficiency of the water is more important.

In practical terms for a sea surface the influence of the thermal inertia of the air column has a smaller effect, for a land surface it needs attention.

Transient thermal analysis of atmosphere/soil/sea of a rotating planet

The SS analysis of the evacuation of heat from the planet can be limited to the analysis of the atmosphere for given surface temperature with corresponding atmospheric temperatures according to the environmental lapse rate, as depicted in figure 2.

The SS stack model as described above and in more detail in [1, 2, 4] is based on the measured temperature profile. The mismatch of incoming absorption and outgoing emission at each level can only be due to mechanisms other than LW-radiation:

convection of sensible and latent heat and absorption of incoming SW sun radiation..

It follows that in SS conditions the evacuation from the surface by radiation into the atmosphere is small due to the fact that the environmental lapse rate keeps the temperature of the lower atmosphere close to the surface temperature.

The radiation model of the stack indicates that surface radiation and absorption thereof

into the atmosphere is small. Natural convection models in a 1-D dimension are not evident: Grasshof numbers need a length parameter and a temperature difference parameter. We come back on that issue in appendix 1.

In the transient analysis the surface temperature is not anymore prescribed but follows from heat balances between the atmosphere and the sub-surface, both for the case of a sea surface and for the case of a soil surface.

The heat transport in the sub-surface is governed by diffusion for which FEM models are standard. See Appendix 1.

In case of a sea surface, waves homogenizes the temperature profile near the surface which can be modeled by artificially giving a higher conductivity in layers close to the surface in the FEM conduction model. During the night, convection keeps the temperature of the surface close to the temperature of the deeper layers.

The diffusion in the air is very low because of the small conduction coefficient.

Nevertheless, we take into account conduction in land/sea as well as in air by means of one and the same FEM model. We add to the nodes of the FEM radiation model [2, 4] the nodes in the sub-surface, in total **Ntot** nodes:

$$\mathbf{Ntot} = \mathbf{nods} + \mathbf{Nze} - 1 \quad (12)$$

A typical value of **Ntot** is 50, 40 in the atmosphere and 10 in the sub-surface space of sea or land. The nodes spacing at the interface is finer as compared to the remaining of the atmosphere and the remaining of the depth of the subsurface. See Appendix 1.

From Appendix 1 we write the transient FEM system of **Ntot** simultaneous equations:

$$\mathbf{M} * \mathbf{dT/dt} + \mathbf{KC} * \mathbf{T} = \mathbf{rhs} \quad (13)$$

M heat capacity matrix **Ntot x Ntot** assembled from the elements in air and in soil/water with components in $J/^{\circ}K/m^2$

KC conductivity matrix **Ntot x Ntot** assembled from the elements in air and in soil/water , with components in $J/^{\circ}K/m^2/sec = W/^{\circ}K/m^2$

T vector of unknown temperature in atmosphere and subsurface space, of length **Ntot**.

rhs right hand side input fluxes due to sun, as well as due to radiation, vector of length NT, with components in W/m^2

The mass matrix has straight forward contributions from air and from the sub-surface, soil or water. See Appendix 1.

The conductivity matrix for air has very low coefficients for the major part of the column of air. Near the surface, the conductivity is artificially given higher values. In case of a sea surface, water elements near the surface are also given higher values of conductivity because of mixing due to surface waves and convection from the warmer lower levels.

The right hand side input fluxes **rhs** consist in a SS simulation of the global and annual mean value of incoming sun power at the top of the atmosphere, with **qtoa**=240. In the transient simulation, the diurnal variation of incoming sun SW radiation, at a latitude where the diurnal mean is 240 W/m², can be written as:

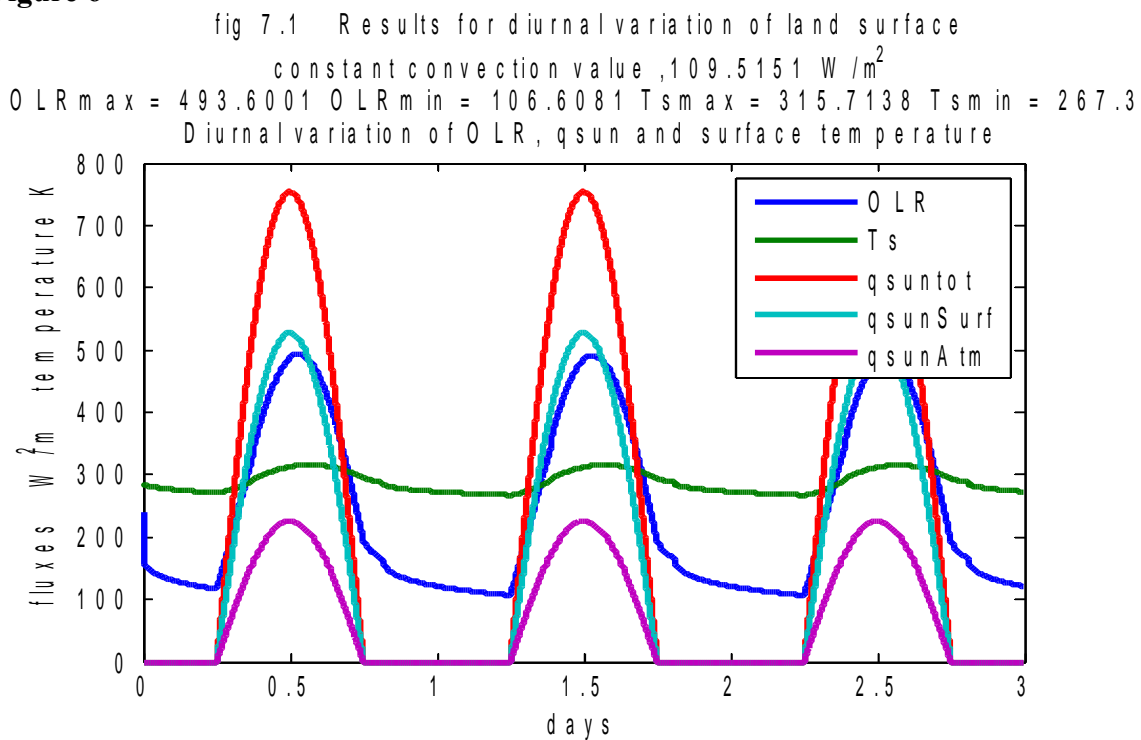
$$q_{sun} = \pi * q_{toa} * \sin(\omega t - \pi/2) \quad \text{with} \quad \omega = 2\pi/24/3600 \text{ rad/sec} \quad (14)$$

$$\text{if } q_{sun} < 0 \text{ then } q_{sun} = 0$$

It means we analyze diurnal variations at a latitude where at March 21 and September 21 the mean daily parameters are equal to the annual and global mean.

Figure 6 shows the daily variation (14) of sun power. The curves OLR and Ts are the result of the transient model we are describing, for a land surface.

Figure 6



In Appendix 1 is discussed how the flux **qsun** is distributed over the nodes in vertical direction in order to take into account that from the **qtoa** = 240 W/m² SW sun flux at top of atmosphere only **qsurfSW** =168 reach the surface and **qatmSW** = 72 is absorbed in the atmosphere, as already discussed in the global and annual mean heat budget, depicted in

figure 1. In Appendix 1 it is also discussed that the 168 arriving at the surface can indeed be allocated to the land surface node.

In case of a sea surface the q_{sun} arriving at the surface has to be distributed over several nodes in the water sub-surface.

The solution of the transient equation (13) is carried out by means of an implicit integration scheme using space-time finite elements, available in text books, on numerical analysis:

$$\Delta T = (\text{inv} (M+2/3*KC * \Delta t)) * (- KC * T + rhs + 2/3 \Delta rhs) * \Delta t \quad (15)$$

$$T = T + \Delta T$$

$$t = t + \Delta t$$

These equations are programmed in a MATLAB program of which the listing is given in Appendix 2. It is an update of the MATLAB program given in [4].

Numerical results for the time-dependent heat evacuation

The scope of this paper is to show that the stack model as presented in [1, 2, 4] is an effective and easy scheme to determine the SS annual mean evacuation of heat from the planet. It turns out that the absorption of the atmosphere is a mere 6 W/m² instead of the 324 as claimed by IPCC authors and others, see figure 1 and [1].

For the transient analysis the system has been extended and includes now the atmosphere and the sub surface space of the planet. We will consider surfaces of land as well as surfaces of water, by an input key in the corresponding MATLAB computer program. The finite element conduction model can be used for bookkeeping of fluxes following from the stack model $q=K*\theta$ as well as for a technique to artificially increase the conductivity in the air near the surface.

The sub-surface has a depth defined by input, deeper for sea as compared to land, subdivided in N_e-1 finite elements and N_e nodes. N_e is an input parameter, and the program takes care of a node distribution based on geometrical series. The program can handle automatically physical properties (heat capacity ρc and conductivity λ) for soil and for water. The latter can be adopted in order to take into account the additional heat transport due to waves and buoyancy in the upper layers of the sea surface.

Material properties of Table 1 and Table 2 are used, but can be changed by the user.

Table 1 Physical properties of water, soil and air. See Wikipedia

	rhoc ρc J/m ³ /°K	lambda λ J/m/sec/°K
water:	4.2e6	0.6 and higher at the surface due to waves
soil:	1.36e6	1.0 depends also on moister
air:	1.2 e3	0.025

Table 2**Properties of a 10 km column of air with variable density**

surface pressure p N/m ²	specific heat c J/kg/ °C	gravitation g m/sec ²	rhoairH pc/g J/m ² /°C
9.81e4	1000	9.81	1e7

We note that the heat capacity of the 10 km air column is equivalent to a column of water of $10^7/4.2e6 = 2.4$ m or a column of soil of $10^7/1.36e6 = 7.4$ m.

Numerical examples for the diurnal variations of temperatures.

We will analyse the diurnal variation of surface and sub-surface temperature distribution of the planet with corresponding variations in heat evacuation.

The SW flux of the sun, both **qsurfSW** at the surface and **qatmSW** absorbed in the atmosphere, including the resulting surface temperature **Ts** and the corresponding outgoing LW radiation **OLR**, for a land surface, have already been depicted in figure 6.

For a sea surface the diurnal variations are less pronounced,

We will deal first with sea surface analyses, which correspond to about 70% of the planet, and next with the detailed results of land/soil analyses.

Analyses for the case of a sea surface

The heat transport in water is not only due to diffusion:

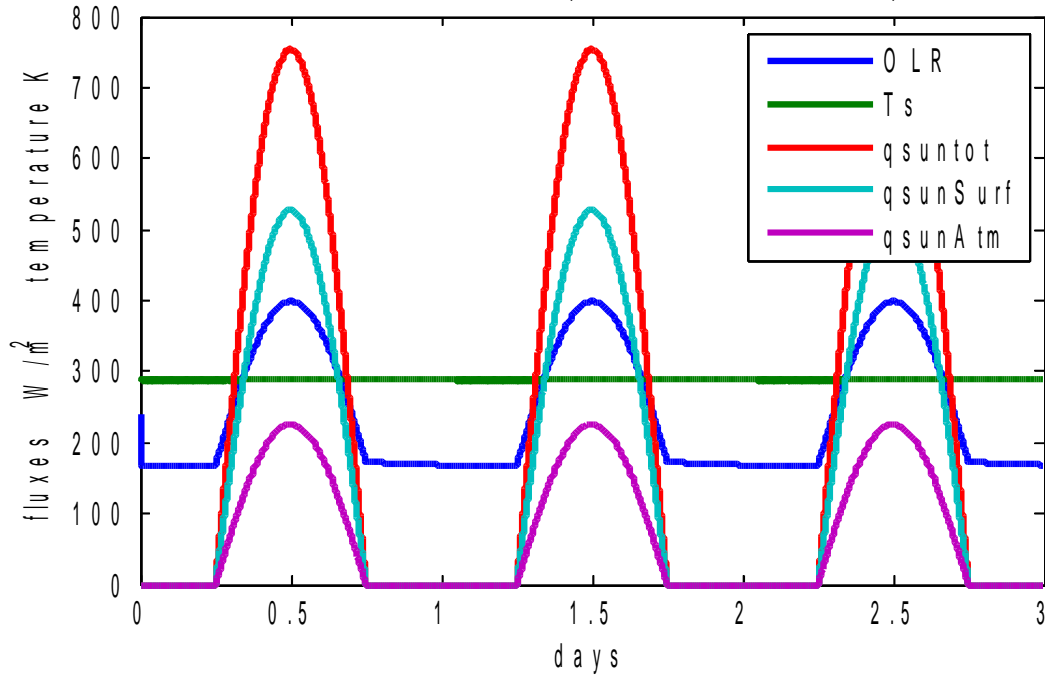
- first of all the incoming sun light is not only absorbed at the surface but within, say, 1 meter depth from the water surface
- the temperature of the upper layers are homogenized due to waves
- as soon as the surface temperature would become colder than the deeper layers, convection tries to homogenize the water temperature.

In the diffusion model as implemented in a MATLAB program the **rhs** vector, as defined in equation (14), takes care of the first point by absorbing the incoming SW sun radiation within the upper layers of the water mass.

In figure 7 the diurnal variations are given for a sea surface.

Figure 7

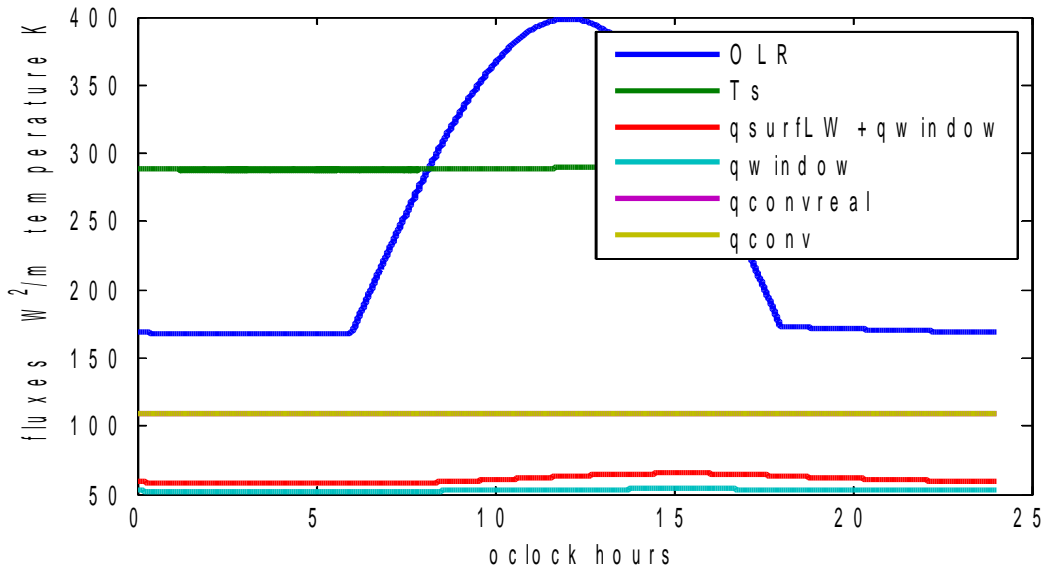
fig 7.1 Results for diurnal variation of sea surface
constant convection value ,109.5151 W /m²
O L R m a x = 398.672 O L R m i n = 167.9248 T s m a x = 290.0456 T s m i n = 287.98
Diurnal variation of O L R , q s u n and surface temperature



In figure 8 the results of figure 7 are repeated in more detail.

Figure 8

fig 7.2 Results for diurnal variation of sea surface
constant convection value ,109.5151 W /m²
O L R m a x = 398.672 O L R m i n = 167.9248 T s m a x = 290.0456 T s m i n = 287.98
Diurnal variation O L R and T s for sea surface



We see that in the atmosphere above a sea surface, the temperatures do not change very much. The OLR changes only because a fraction (72/240) of SW sunlight is absorbed in the atmosphere.

In figure 9 and 10 are given the sub-surface maximum and minimum temperature of the sea water respectively of the atmosphere.

Figure 9

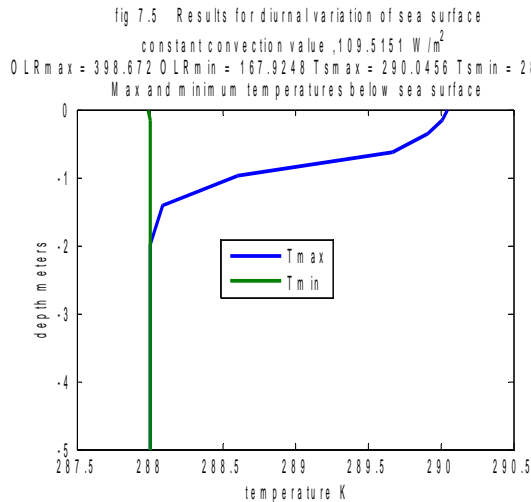
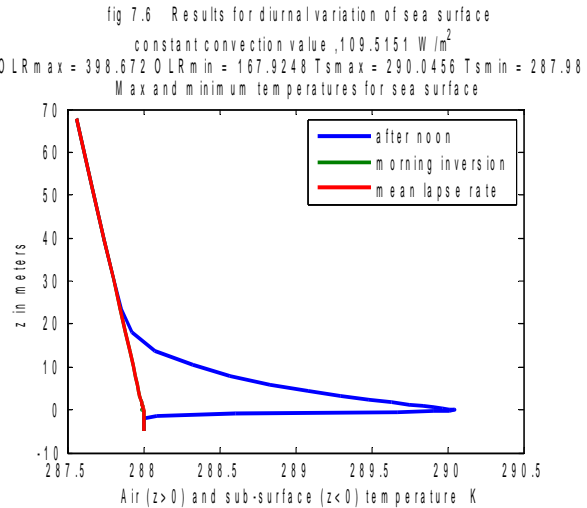


Figure10



We see in figure 9 that the upper 1 meter of the sea surface has a block diurnal variation of 2 K. This is obtained by the FEM diffusion program by artificially given a high conductivity to take into account the homogenization due to surface waves, as well as by letting the SW sun light penetrate the upper layers to a depth of 0.6 m.

The FEM program is taking care of the bookkeeping.

Figure 10 gives the diurnal variation of the lower layers of the temperature above the sea water.

Analyses for the case of a land surface

The planet earth has a surface of which 70% is sea. In the foregoing section we have seen that the thermal inertia of the upper layers of the sea keeps the diurnal variation of temperatures low.

It seems that in ancient Egypt one was able to make ice merely because of the diurnal variation of the surface temperature in the desert belt. We will show that it is possible.

The results for the case of a land surface were already given in figure 6.

We repeat them in more detail in figure 11.

We see that the diurnal variation of the surface temperature is between 316 and 267 K and it is indeed a confirmation that in ancient Egypt the ice could be made overnight!

We draw further the attention on the difference between **qwindow** and **qsurfLW**.

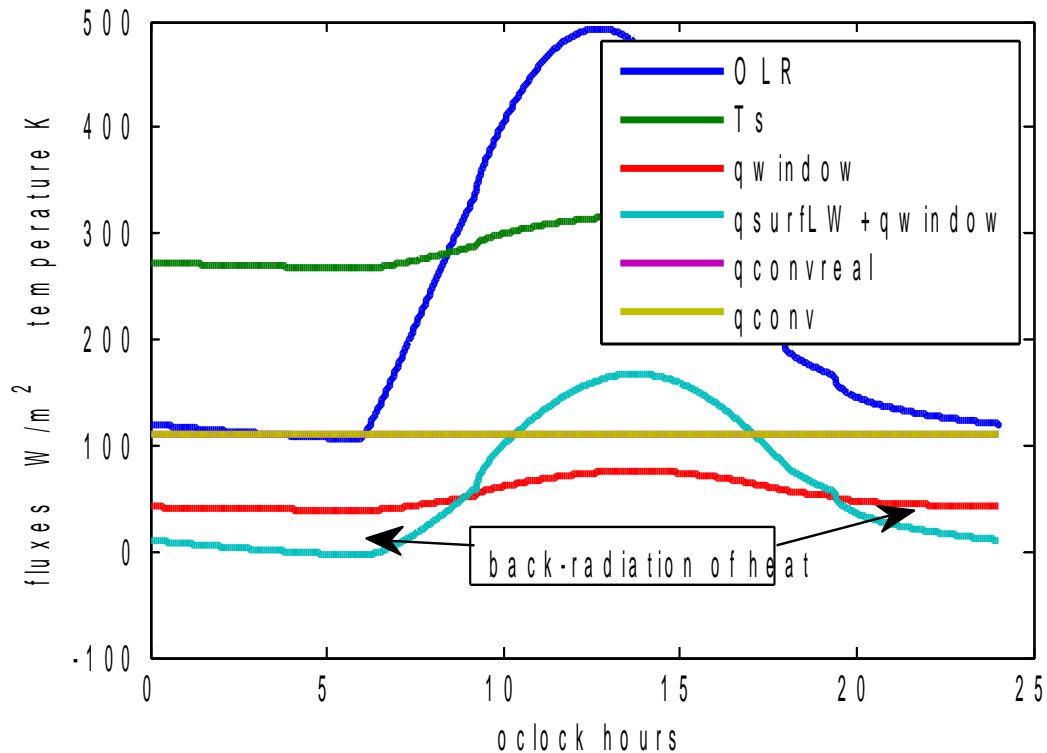
As seen from figure 5 this difference was about **qatmLW0** = 6 Watt/m², for different surface temperatures but in SS conditions.

In the transient case the difference is bigger and positive during the day, but negative

during the night. Indeed during the night the **warmer** atmosphere is sending heat to the **colder** surface, back-radiation of heat!

Figure 11

fig 7.2 Results for diurnal variation of land surface
 convection remains constant to SS value 109.5151 W/m^2
 $\text{OLR}_{\text{max}} = 495.5792$ $\text{OLR}_{\text{min}} = 105.5341$ $T_{\text{smax}} = 316.0464$ $T_{\text{smin}} = 267.1$
 Diurnal variation OLR and Ts for land surface



The outgoing long wave radiation OLR is given by:

$$\text{OLR} = q_{\text{window}} + q_{\text{atmLW}} + q_{\text{atmSW}} + q_{\text{conv}} \quad (16)$$

The first three terms are of the radiation type. It are fast components, they follow from the radiation stack model $q = K \cdot \theta$.

The convection term is slower. Here it is supposed that the value remains constant.

In Appendix 1 the term is discussed in more detail with numerical examples.

The temperature of the sub-surface is defined by the FEM conduction model, as well as for the near surface air temperatures. The sub-surface maximum and minimum temperatures and the maximum and minimum near surface atmospheric temperature profiles are given in figure 12 respectively figure 13.

Figure 12

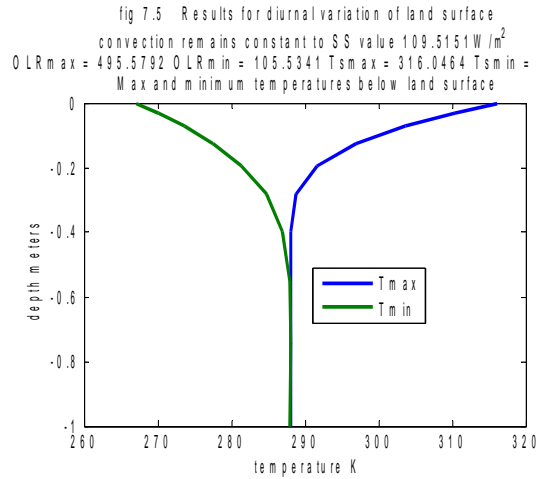
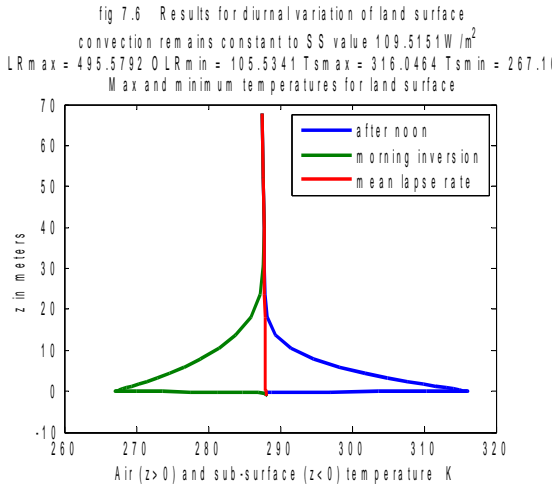


Figure 13



We see a strong diurnal variation between 267 and 316 K, a variation of about 50 °C. These results confirm that in ancient Egypt one was able to make ice! The analyses do not take into account diurnal variations of the absorption coefficients due to fog. The studies in Appendix 1 show that in case the heat flux due to convection increases during the day and decreases during the night, the diurnal variations are less pronounced.

Conclusion

The numerical results show clearly that the transient version of the stack model gives coherent results for OLR and surface temperatures for a strong diurnal variation of the sun intensity. In the model an eventual variation of the absorption coefficient as function of the atmospheric temperatures has not been taken into account.

The scope of this paper is not to give detailed results for diurnal variations of the sun power, but rather to demonstrate that 1-D SS models based on the one-way heat flow concept of Claes Johnson are an accurate tool to show the very small influence of IR-active gases for the global and annual mean heat budget of the planet.

In figure 1, taken from [1, 2, 4] where the implementation of the one-way heat flow finite element model has been described in detail, it has been shown that the evacuation of heat from the planet surface in SS is not by radiation but rather by convection.

In diurnal transients radiation has more effect.

Radiation is of course also in SS conditions the mechanism to evacuate the heat to outer space from higher levels of the atmosphere by means of the IR-active gases with 3 or more atoms per molecule.

It has been shown in [2, 4] that doubling the concentration of the IR-active CO₂ from 0.04% to 0.08% is causing a mere 0.03°C increase in surface temperature: the so-called CO₂ sensitivity.

Acknowledgement

The author is indebted to Claus Johnson who inspired him to implement the one-way heat flow finite element models.

The help of Hans Schreuder to edit this paper, to organize a peer review and host it on his site is acknowledged.

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- [5] <http://claesjohnson.blogspot.fr/>
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Appendix 1

Heat capacity matrix

For an element with 2 nodes, size Δz meter and heat capacity ρc J/m³/K the 2x2 heat capacity element matrices become:

$$\mathbf{m} = \rho c \Delta z / 6 \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} \quad (\text{A1.1})$$

These element matrices are assembled in the classical way of FEM to give a tri-diagonal mass matrix \mathbf{M} .

Conductivity matrix

With heat conduction coefficient λ J/m/sec/K the 2x2 heat conductance element matrices become:

$$\mathbf{kc} = \lambda \Delta z \begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix} \quad (\text{A1.2})$$

These element matrices are assembled in the classical way of FEM to give a tri-diagonal conductivity matrix \mathbf{KC} .

Interaction between sea or land with atmosphere

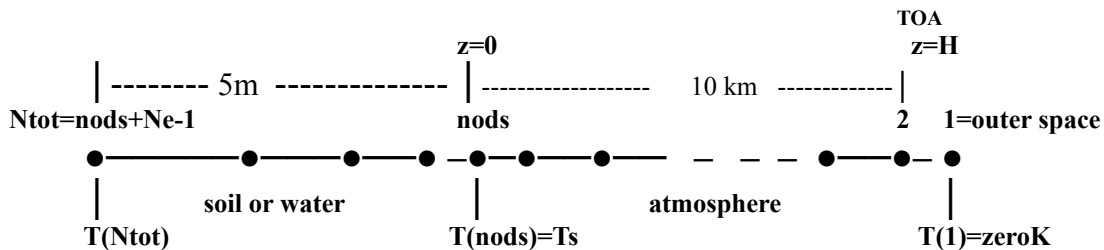
The FEM model of the sub-surface and the atmosphere is depicted in figure A1.2

Figure A1.2 Finite element model for conduction in sub-surface and atmosphere

$$\mathbf{M} * d\mathbf{T}/dt + \mathbf{KC} * \mathbf{T} = \mathbf{rhs} \quad (13) \text{ or } (A1.3)$$

\mathbf{M} and \mathbf{KC} are matrices of order $N_{tot} \times N_{tot}$ (typically 50 x50)

\mathbf{T} and \mathbf{rhs} are vectors of order N_{tot}



We see in the atmosphere the same number **nods** as in the radiation FEM model of [2], $\mathbf{q} = \mathbf{K} * \boldsymbol{\theta}$, only the numbering has been inverted in order to obtain banded heat capacity and conductivity matrices, which in MATLAB are dealt with more efficiently.

Typical values are **nods**=40, **Ne**=10 which gives $N_{tot} = 51$. In the MATLAB program of which the listing is given in Appendix 2, the number of nodes is given by input. The generation of the mesh is carried out automatically by means of geometrical series with a finer distribution of nodes near the intersection of the surface of the planet (sea or land) and the atmosphere.

The contribution of the conductivity of the atmosphere is very small, only near the surface conduction gives a contribution. The heat transport from the surface, at least for SS conditions, is mainly by convection, $\mathbf{q}_{conv} = 109 \text{ W/m}^2$ for the annual and global mean heat budget) and 59 by LW radiation of which $\mathbf{q}_{window} = 53$ through the atmospheric window and LW radiation absorbed by the atmosphere, $\mathbf{q}_{atmLW} = 6$, These figures have already been given in figure 1 and in figure 4, which is a graphical display of the stack model: $\mathbf{q} = \mathbf{K} * \boldsymbol{\theta}$.

The scope of the transient model is to be able to find fluxes and temperatures for a varying diurnal sun power, as given in equation (14) of the main text and repeated here for convenience:

$$\mathbf{q}_{sun} = \pi * \mathbf{q}_{toa} * \sin(\omega t - \pi/2), \quad \omega = 2\pi/24/3600 \text{ rad/sec} \quad (A1.4) \text{ or } (14)$$

$$\text{if } \mathbf{q}_{sun} < 0 \text{ then } \mathbf{q}_{sun} = 0$$

We note that the average daily value is $\mathbf{q}_{toa} = 240 \text{ W/m}^2$. For the SS we define the following fluxes.

First of all the mean sun power does not arrive completely at the surface as already indicated in figure 1: out of the mean 240 W/m² SW sun radiation only 168 arrive at the surface and 72 are absorbed in the atmosphere.

We call them **qsurfSW0** respectively **qatmSW0**, where 0 stands for **SS**.

These values are defined for a temperature distribution defined by a surface temperature **Ts=288 K**, an environmental lapse rate **ELR = -6.5 K/km**, a constant sun power **qsun**, see figure 2. (NB albedo is included in the global and annual mean value for **qsun = 240**)

$$\mathbf{qsurfSW} = \mathbf{qsun/ORLO*qsurfSW0} \quad (\mathbf{A1.5})$$

$$\mathbf{qatmSW} = \mathbf{qsun/OLR0*qatmSW0} \quad (\mathbf{A1.6})$$

$$\mathbf{qsurfLW} = \mathbf{q(1)} \quad (\mathbf{A1.7})$$

$$\mathbf{OLR} = \mathbf{-qnods)} \quad (\mathbf{A1.8 \text{ only SS}})$$

$$\mathbf{qconv} = \mathbf{OLR - qsurfLW - qatmSW} \quad (\mathbf{A1.9 \text{ only SS}})$$

$$\mathbf{qwindow} = \mathbf{epsilon*sigma*Ts^4} \quad (\mathbf{A1.10})$$

$$\mathbf{qatmLW} = \mathbf{qsurfLW - qwindow} \quad (\mathbf{A1.11})$$

For situations where the sun power varies, the relations for heat fluxes depending on radiation expressed as function of the temperature related variables **θ**, remain more or less the same since radiation is a fast mechanism.

The heat flux due to convection is a slower mechanism!

Indeed **(A1.9 only SS)** has been obtained from the condition of “heat in = heat out” at each level of the stack and such a balance is only valid in **SS** conditions.

In conclusion: equation **(A1.8 only SS)** for the outgoing LW radiation **OLR**, is not valid for transients and equation **(A1.9 only SS)** should be written as:

$$\mathbf{OLR} = \mathbf{qsurfLW + qatmSW + qconv} \quad (\mathbf{A1.12 \text{ transient}})$$

In this latter equation the convection term **qconv** is unknown!

Convection in transient analysis

The stack-model **q = K*θ** is a radiation model which showed that in **SS** condition, radiation is not the main mechanism for the evacuation of heat from the surface of the planet to higher layers. The missing 109 W/m² in the balance of incoming and outgoing radiation at the various layers are allocated to convection of sensible and latent heat from the surface.

A comparison has been made in [1] where IPCC authors gave the convection terms. Unfortunately in those IPCC K&T diagrams one has inserted the non-physical back-radiation, which seems to be a habit in astrology.

In the stack model, the term **qconv** in SS conditions is known, as well as for very slow seasonal variations where the atmospheric temperature follows a translation of the profile of the environmental lapse rate. It is the case 3 temperature variation as given in figure 5, a SS solution for different atmospheric temperatures with one and the same lapse rate. We see in figure 5, a slight increase of that term **qconv** with the surface temperature, as could be expected for a natural convection phenomena.

Natural convection is analysed by dimension analysis involving Grasshof numbers. But Grasshof numbers need a length parameter which in cities could be the height of the buildings: sailplane pilots use the thermals initiated from high buildings like churches. In a general model with a flat land surface and a flat sea surface we cannot identify a length parameter. We are therefore obliged to make hypotheses:

- (1) we take the **qconv** term constant for diurnal variations of sun power
- (2) we take the **qconv** for the SS of case 3 temperature distribution given in figure 5
- (3) we take the **qconv** proportional to **qsun** with an average value to the annual and global mean

Moreover, we introduce inertia by means of a time constant of about 2 hours, which is based on the experience of sail plane pilots: thermals start in May by 10 o'clock in the morning.

When it turns out that the results of the two approaches are close, than we can have confidence in them.

Ad 1

The first hypothesis is straight forward to implement:

$$\mathbf{qconv} = 109, \text{ as follows from the SS radiation model } \mathbf{q} = \mathbf{K} * \mathbf{\theta} \quad (\mathbf{A1.13})$$

In the main text the results of this hypothesis for the diurnal variation of temperatures and OLR for a constant convection have been presented.

Ad2

The second hypothesis gives by means of a linear regression from figure 5:

$$\mathbf{qconv} = 1.5791 * (\mathbf{T_s} - \mathbf{T_s0}) \quad (\mathbf{A1.14})$$

Ad3

The third hypothesis is with the convection term proportional to the surface sunpower:

$$\mathbf{qconv} = \mathbf{qconv0} * \mathbf{qsurfSW} / \mathbf{OLR0}$$

In transient analyses for a land surface we will use these relations for the contribution by convection of heat from the surface of the planet to upper layers of the atmosphere.

For sea surface the variation of \mathbf{qconv} is small.

For a sea surface hypothesis 1 is to be preferred.

Time delay for the establishment of the convection

The heat flux by convection is not established immediately and a delay mechanism with a time constant τ has been implemented:

$$\tau * d\mathbf{qconvreal} / dt + \mathbf{qconvreal} = \mathbf{qconv} \quad (\mathbf{A1.15})$$

In the transient analysis of the system of \mathbf{Ntot} (typical 50) simultaneous equations (**14**) we add this equation to follow up $\mathbf{qconvreal}$ using a space-time finite element algorithm:

$$\Delta(\mathbf{qconvreal}) = 1 / (1 + 2/3 * \Delta t / \tau) * (-\mathbf{qconvreal} + \mathbf{qconv} + 2/3 * \Delta \mathbf{qconv}) * \Delta t / \tau \quad (\mathbf{A1.16})$$

$$\mathbf{qconvreal} = \mathbf{qconvreal} + \Delta(\mathbf{qconvreal})$$

$$\mathbf{t} = \mathbf{t} + \Delta \mathbf{t}$$

The result of this delay is that convection starts to rise a bit later after sun rise and continues a bit after sun set.

Such a delay is also introduced by the thermal inertia of the surface.

Numerical results with a diurnal variation of convection

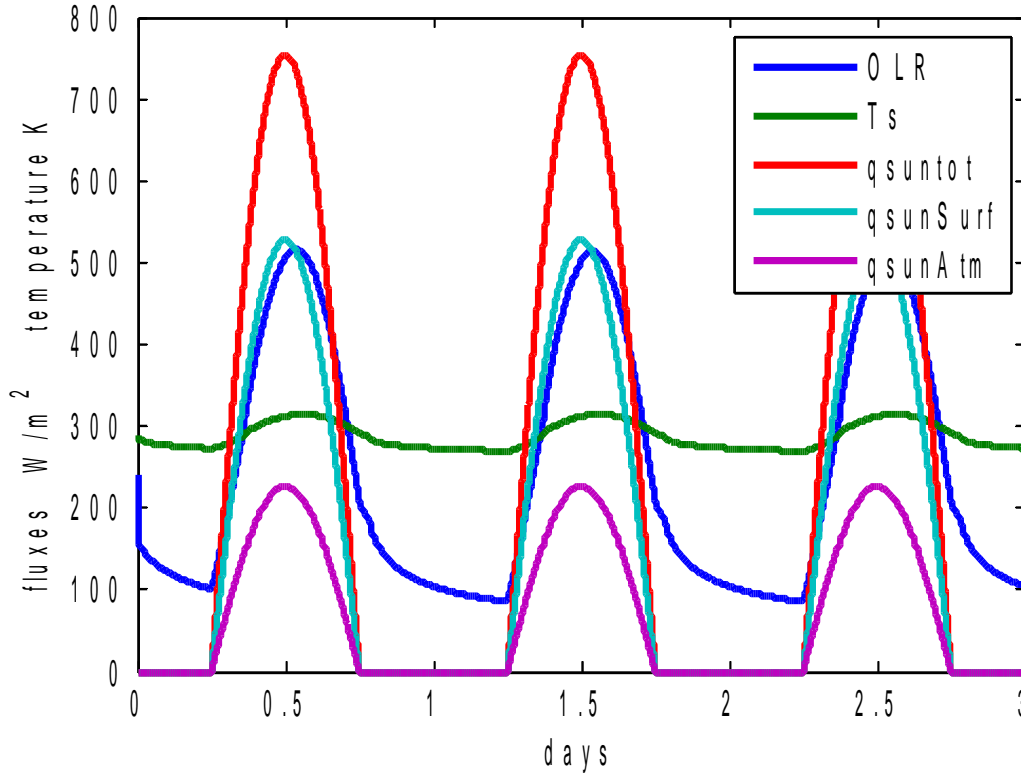
We repeat the analyses for a land service from the main text but let the convection heat flux vary according to the hypothesis 2, as given in (**A1.14**) with a time delay of 2 hours according to (**A1.15**).

In figure **A1.2** are given the diurnal variation of sun power and the corresponding **OLR** and surface temperature for a simulation of 3 days.

The time constant for the delay of the heat flux due to convection is 2 hours.

Figure A1.2

fig 7.1 Results for diurnal variation of land surface
 linear regression $q_{conv} = 1.5791 * (T_s - 288)$, time delay 2 hr
 $O L R_{max} = 516.1156$ $O L R_{min} = 85.5977$ $T_{smax} = 314.4812$ $T_{smin} = 269.62$
 Diurnal variation of O L R, q_{sun} and surface temperature

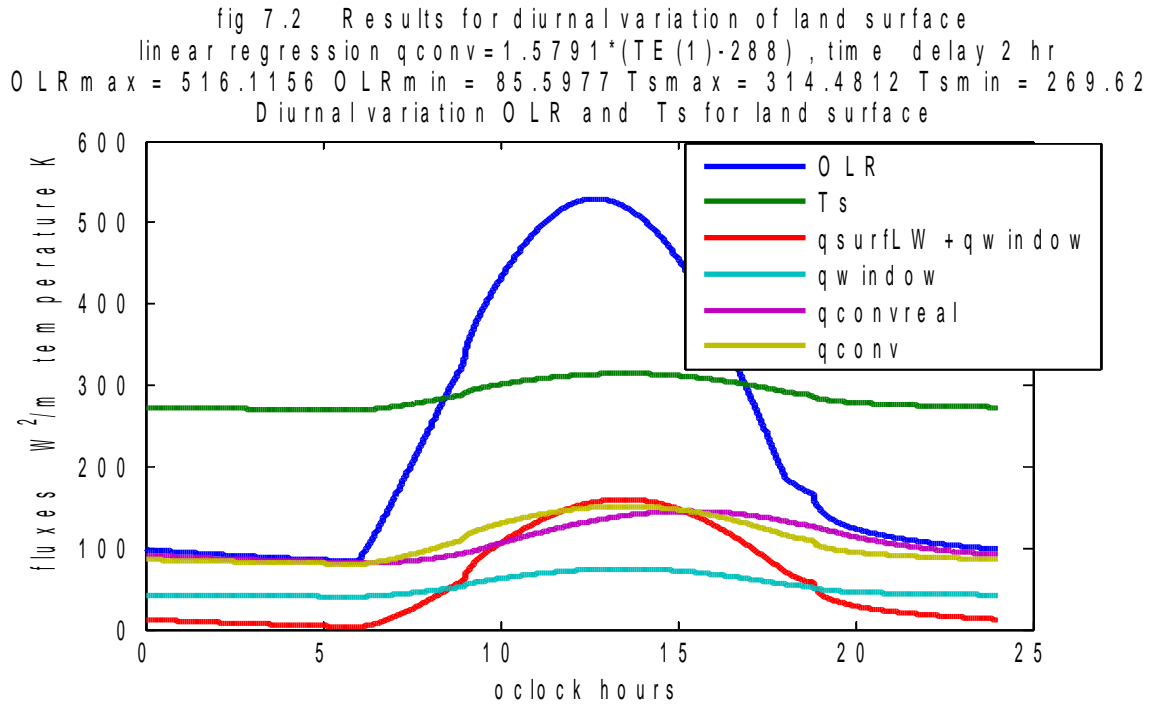


The time constant for the delay of the heat flux due to convection is 2 hours. Such simulations for 3 days are used to confirm stationary conditions. For a model with 50 nodes giving rise to 50 simultaneous equations and a time step of 0.01 hour the simulation takes 3 seconds on a PC

In figure A1.3 the more detailed results, covering one single day are given, including the diurnal variation of the heat flux due to convection as shown by the curve **qconv** and the curve **qconvreal**, shifted to the right showing the 1 hour delay.

The convection heat flux shows a diurnal variation from **50 W/m^2** during the early morning just before sun rise to **216 W/m^2** at 14h30, as compared to the constant SS value of **109 W/m^2** in figure 11 of the main text.

Figure A1.3



The diurnal variation of **qconv** affects the diurnal variation of **OLR**.
 The convection contribution is added to the other fluxes influencing the **OLR**:
 The fact that the surface temperature during day time is increased as compared to the global and annual mean value, the outgoing surface heat flux is increased:

- due to the increased LW flux through the window : **qwindow**
- due to the increased LW flux absorbed in the cooler atmosphere: **qatmLW**
- due to the increased convection of heat : **qconv**

and not from the surface but direct into the atmosphere

- sun SW absorption in day time: **qatmSW**

During the night things are reversed.

Starting at sun set at 18h00 until sun rise at 06h00, the warmer atmosphere radiates heat back to the colder surface. The convection heat flux during night is not any more 109 W/m² but is decreased to 80, while at noon until 14h00 it becomes 145.

Figure **A1.3** with the hypothesis **2** for the convection has to be compared to figure **11** for the hypothesis 1 that the convection during night remains at the day time value.

The maximum and minimum temperature profiles of the sub-surface and the maximum and minimum near surface atmospheric temperature profiles are given in figure **A1.4** and figure **A1.5** respectively.

These figures have to be compared with figures **12** and **13**, using hypothesis 1 with a

constant value of 109 W/m^2 during day and night for the heat evacuation by convection. In figures 12 and 13 the diurnal variation of the surface was between 267 and 316 K, while with hypothesis 2 in figures A1.4 and A1.5 the diurnal variation is lower, from 269.6 to 314.5 K.

Figure A1.4

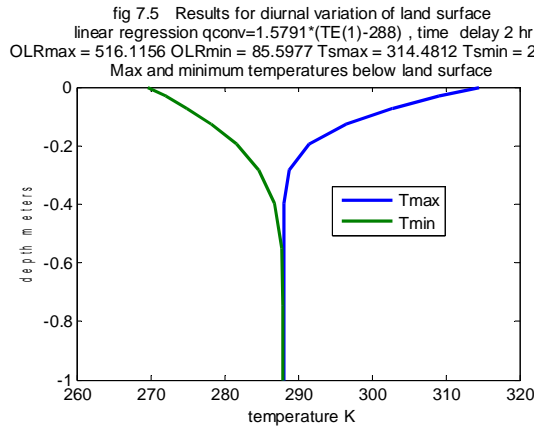
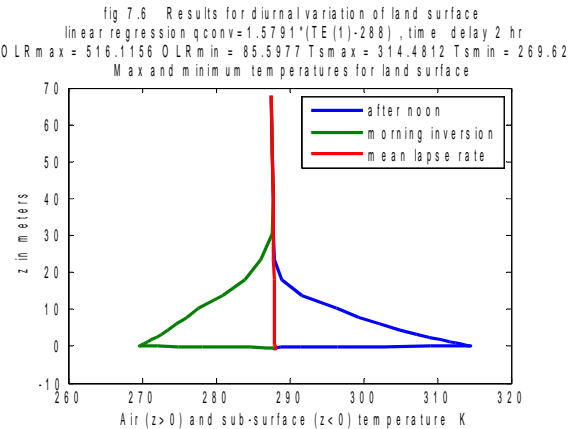


Figure A1.5



Due to the diurnal variation of the convection, according to hypothesis 2, the diurnal variation of the surface temperature has decreased. During day time, more heat travels by convection to upper layers to be emitted by IR-active gases, and during the night less heat is evacuated due to a decrease in convection, as compared to the global and annual mean.

For hypothesis 3 the convection term during the night becomes zero, because it is supposed to be proportional to the surface sun power.

From the numerical results in the main text and in this Appendix we might conclude that the two hypotheses concerning the convection give results close to each other.

The claim of this paper is indeed not to have given precise results of the diurnal temperature and flux variation, but rather to show that the stack-model characteristics embedded in the matrix relation $\mathbf{q}=\mathbf{K} \cdot \boldsymbol{\theta}$, provide coherent input to deal in a transient model with convection without the cumbersome Navier-Stokes equations.

The listing of the MATLAB program with “green “ comments at nearly every line is given in Appendix 2. It is an update of an earlier listing in [4].

Appendix 2

Since October 2014 the MATLAB listing is in a separate paper:

<http://www.tech-know-group.com/papers/Reynen-MATLAB-listing.pdf>

